FMPC 10

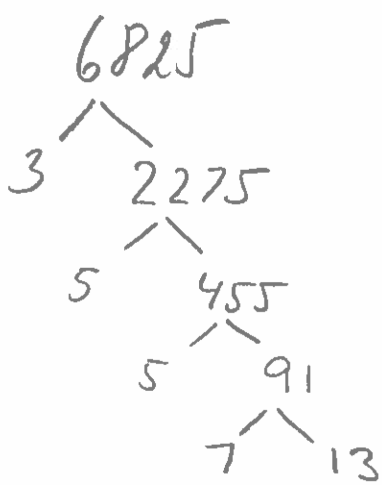
Answers

The Fundamental Theorem of Arithmetic

The Fundamental Theorem of Arithmetic states that every positive integer, except the number one, can be represented in exactly one way apart from rearrangement as a product of a one or more prime numbers.

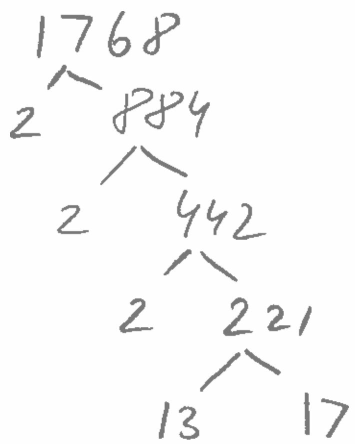
This theorem is sometimes referred to as The Unique Factorization Theorem.

Example 1: Express 6825 as a product of its prime factors.



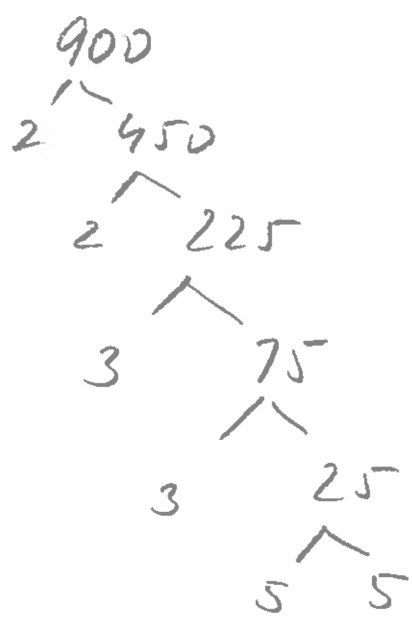
$\therefore 6825 = (3)(5^2)(7)(13)$

Example 2: Prime factorize 1768.



$\therefore 1768 = (2^3)(13)(17)$

Example 3: Prime factorize 900.

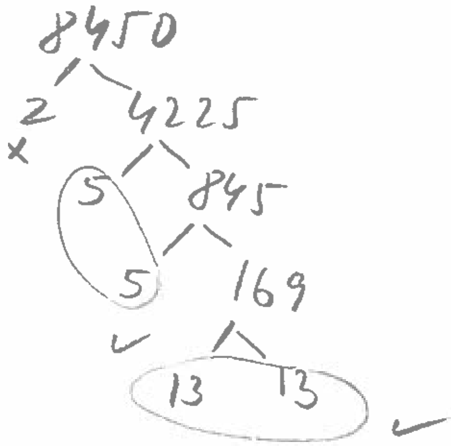


$\therefore 900 = (2^2)(3^2)(5^2)$

APPLICATION OF PRIME FACORIZATION

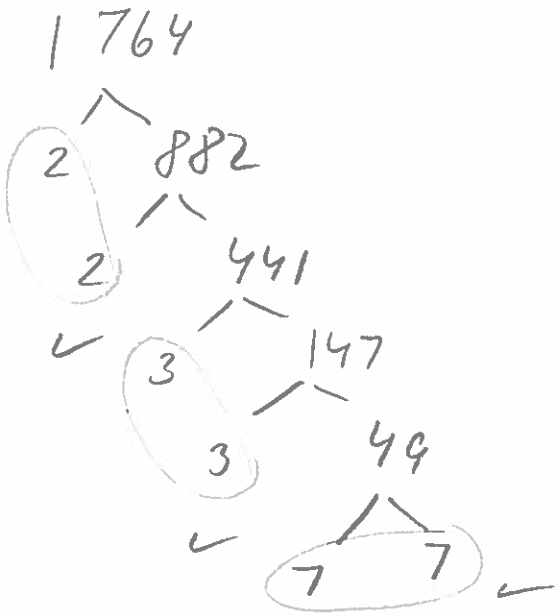
A: Using prime factorization to determine whether a given number is a (perfect) square (number).

Example 1: Determine whether 8450 is a perfect square. Justify your answer.



∴ 8450 is NOT a square number.

Example 2: Determine whether 1764 is a perfect square number. Justify your answer.



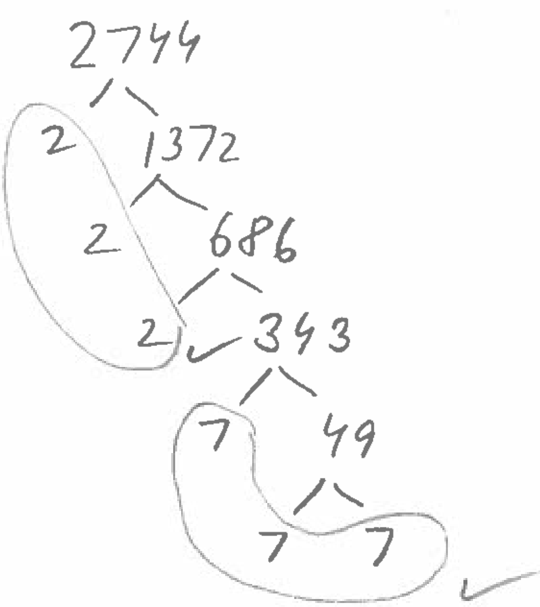
∴ 1764 is a square number because $1764 = (2^2)(3^2)(7^2)$
$= (42)^2$.

B: Using prime factorization to determine whether a given number is a cube (number).

Note: A cube number can be expressed as a product of exactly three identical factors.
Examples of cube numbers:

Power	Number
0^3	0
1^3	1
2^3	8
3^3	27
4^3	64
5^3	125
6^3	216
7^3	343
8^3	512
9^3	729
10^3	1000

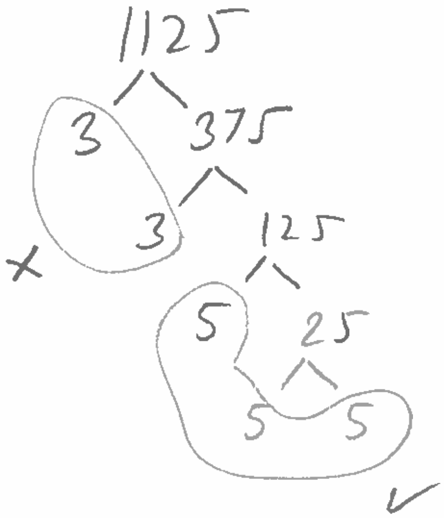
Example 1: Determine whether 2744 is a cube number. Justify your answer.



$\therefore$ 2744 is a cube number because

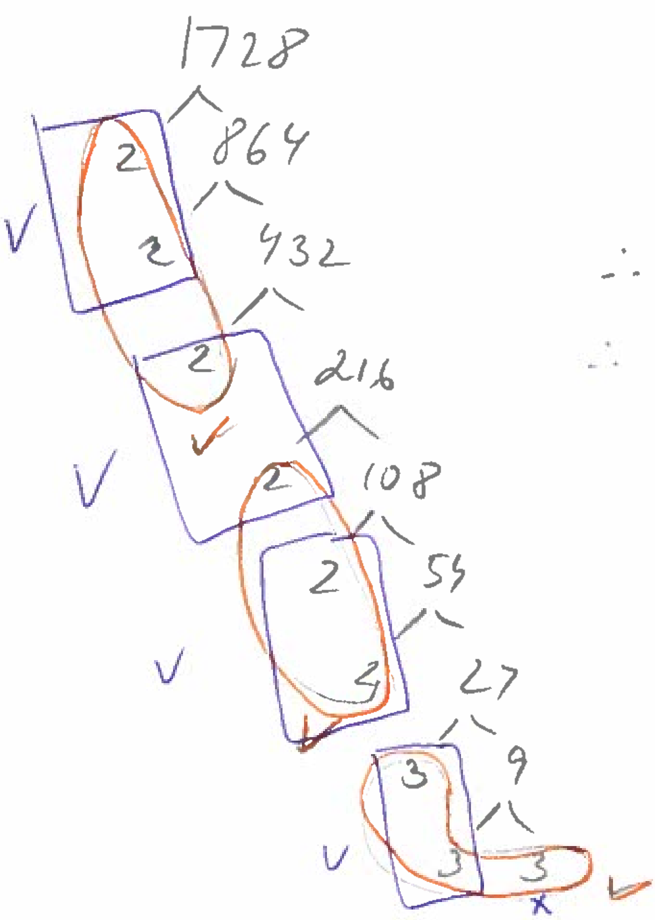
$2744 = (2^3)(7^3) = (2 \cdot 7)^3 = \underline{14^3}$

Example 2: Determine whether 1125 is a cube number. Justify your answer.



$\therefore$ 1125 is not a cube number.

Example 3: Determine whether 1728 is a cube or a square number. Justify your answer.

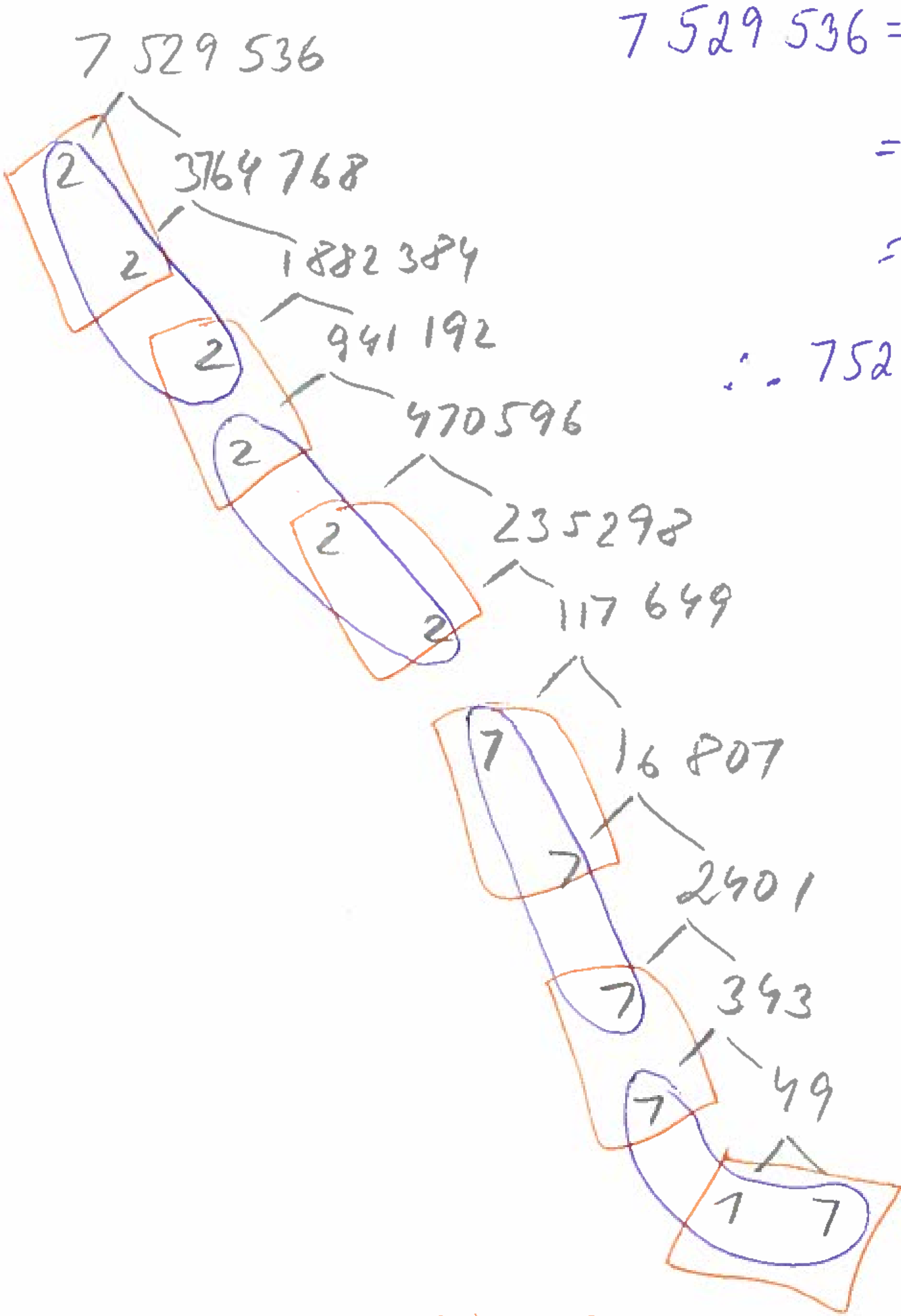


$$1728 = (2^3)(2^3)(3^3) = (2\cdot 2\cdot 3)^3$$
$$= 12^3$$

$\therefore$ 1728 is a cube number

$\therefore$ 1728 is not a square number.

Example 4: Determine whether 7 529 536 is a cube or a square number. Justify your answer using prime factorization.



$7\ 529\ 536 = (2^3)(2^3)(7^3)(7^3)$

$= (2 \cdot 2 \cdot 7 \cdot 7)^3$

$= 196^3$

$\therefore$ 7529 536 is a cube number.

$7\ 529\ 536 = (2^2)(2^2)(2^2)(7^2)(7^2)(7^2)$

$= (2 \cdot 2 \cdot 2 \cdot 7 \cdot 7 \cdot 7)^2$

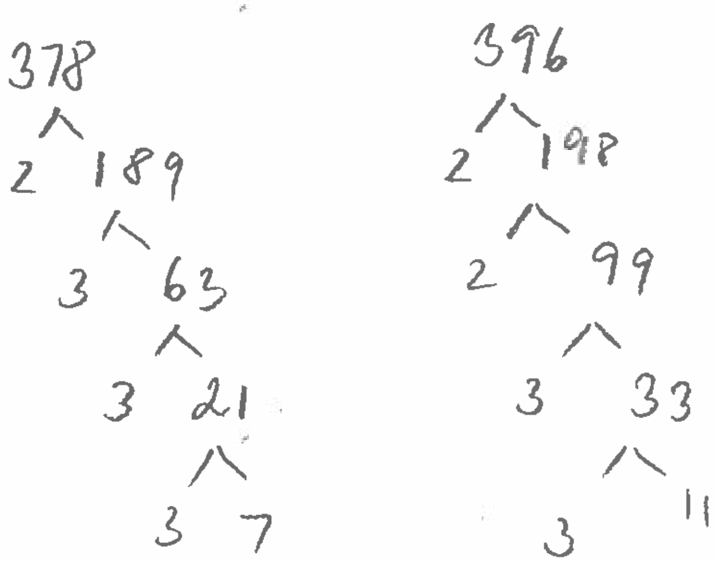
$= 2744^2$

$\therefore$ 7529 536 is a square number

C: Using prime factorization to determine the least common multiple (LCM) of a pair or a group of numbers

Note: It is very useful to know the least common multiple of a group of numbers in order to find a common denominator needed to add and/or subtract fractions.

Example 1: Find the least common multiple (LCM) of 378 and 396.



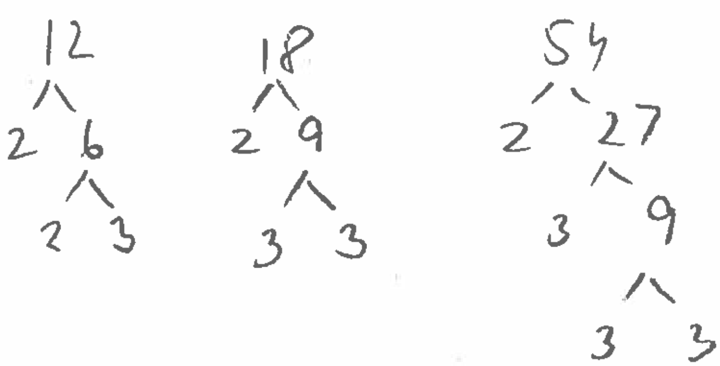
$378 = (2)(3^3)(7)$

$396 = (2^2)(3^2)(11)$

$\therefore \operatorname{lcm}(378, 396) = (2^2)(3^3)(7)(11)$

$= \underline{\underline{8316}}$

Example 2. Find the least common multiple (LCM) of 12, 18 and 54.



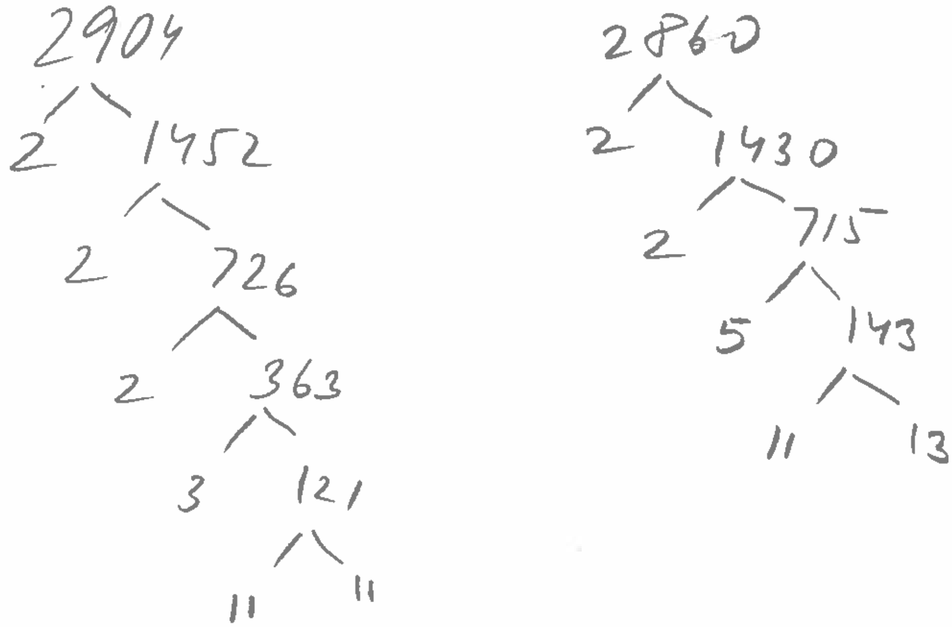
$12 = (2^2)(3)$

$18 = (2)(3^2)$

$54 = (2)(3^3)$

$\therefore \operatorname{lcm}(12, 18, 54) = (2^2)(3^3) = (4)(27) = \underline{\underline{108}}$

Example 3: Find the least common multiple (LCM) of 2904 and 2860.



$$2904 = (2^3)(3)(11^2)$$

$$2860 = (2^2)(5)(11)(13)$$

$$\therefore \operatorname{lcm}(2860, 2904) = (2^3)(3)(5)(11^2)(13)$$

$$= \underline{\underline{188\,760}}$$

D: Using prime factorization to determine the greatest common factor (GCF) of a pair or a group of numbers

Note: It is very useful to know the greatest common factor of a pair of a group of numbers when working with fractions because the greatest common factor can be factored out and the fractions can be simplified.

Note: If numbers do not have a prime factor in common, their gcf = 1

Example 1: Find the greatest common factor (GCF) of 285 and 5445.

285
3 95
5 19

5445
3 1815
3 605
5 121
11 11

$285 = 3 \times 5 \times 19$

$5445 = 3 \times 3 \times 5 \times 11 \times 11$

$\therefore gcf(285, 5445) = (3)(5) = 15$

Example 2: Find the greatest common factor of 234 and 147.

234
2 117
3 39
3 13

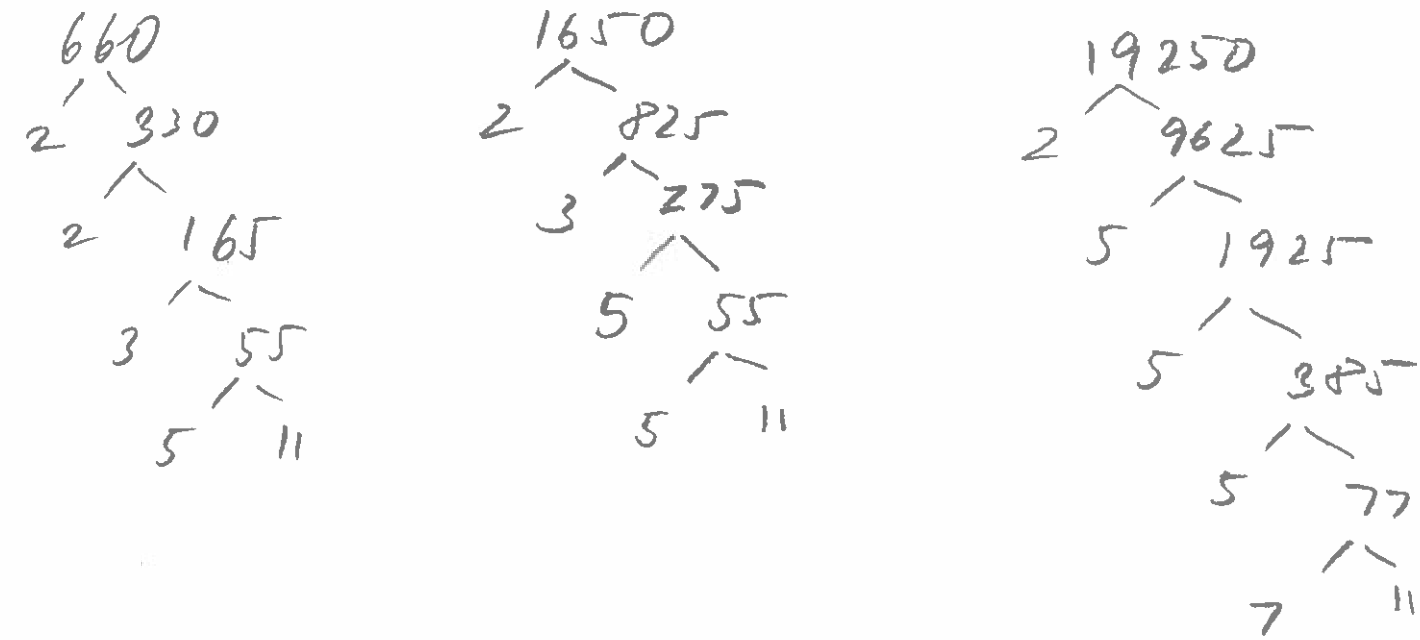
147
3 49
7 7

$234 = 2 \times 3 \times 3 \times 13$

$147 = 3 \times 7 \times 7$

$\therefore gcf(147, 234) = 3$

Example 3: Find the greatest common factor of 660, 1650 and 19250.



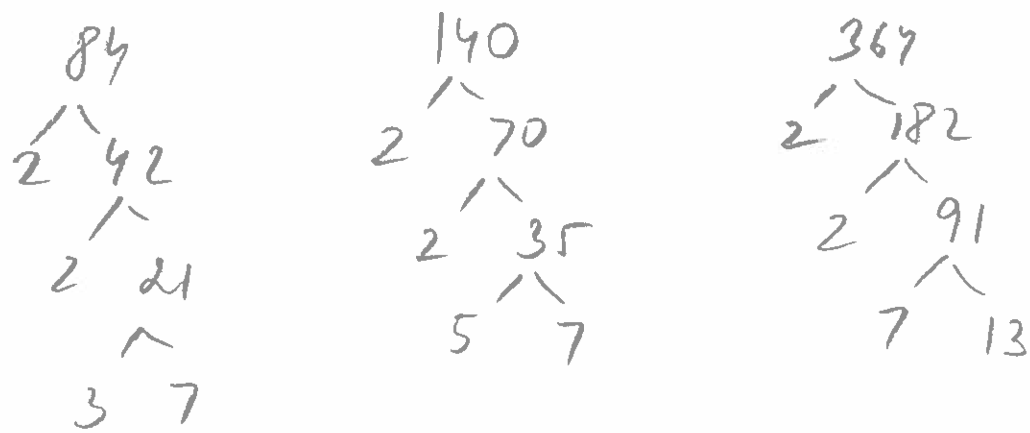
$660 = 2 \times 2 \times 3 \times 5 \times 11$

$1650 = 2 \times 3 \times 5 \times 5 \times 11$

$19250 = 2 \times 5 \times 5 \times 5 \times 7 \times 11$

$\therefore gcf = (2)(5)(11) = \underline{\underline{110}}$

Example 4: Find the greatest common factor (GCF) of84, 140 and 364.



$$84 = 2 \times 2 \times 3 \times 7$$
$$140 = 2 \times 2 \times 5 \times 7$$
$$364 = 2 \times 2 \times 7 \times 13$$

$$\therefore \ gcf(84, 140, 364) = 2 \times 2 \times 7$$
$$= \underline{\underline{28}}$$