Notes

FMPC10

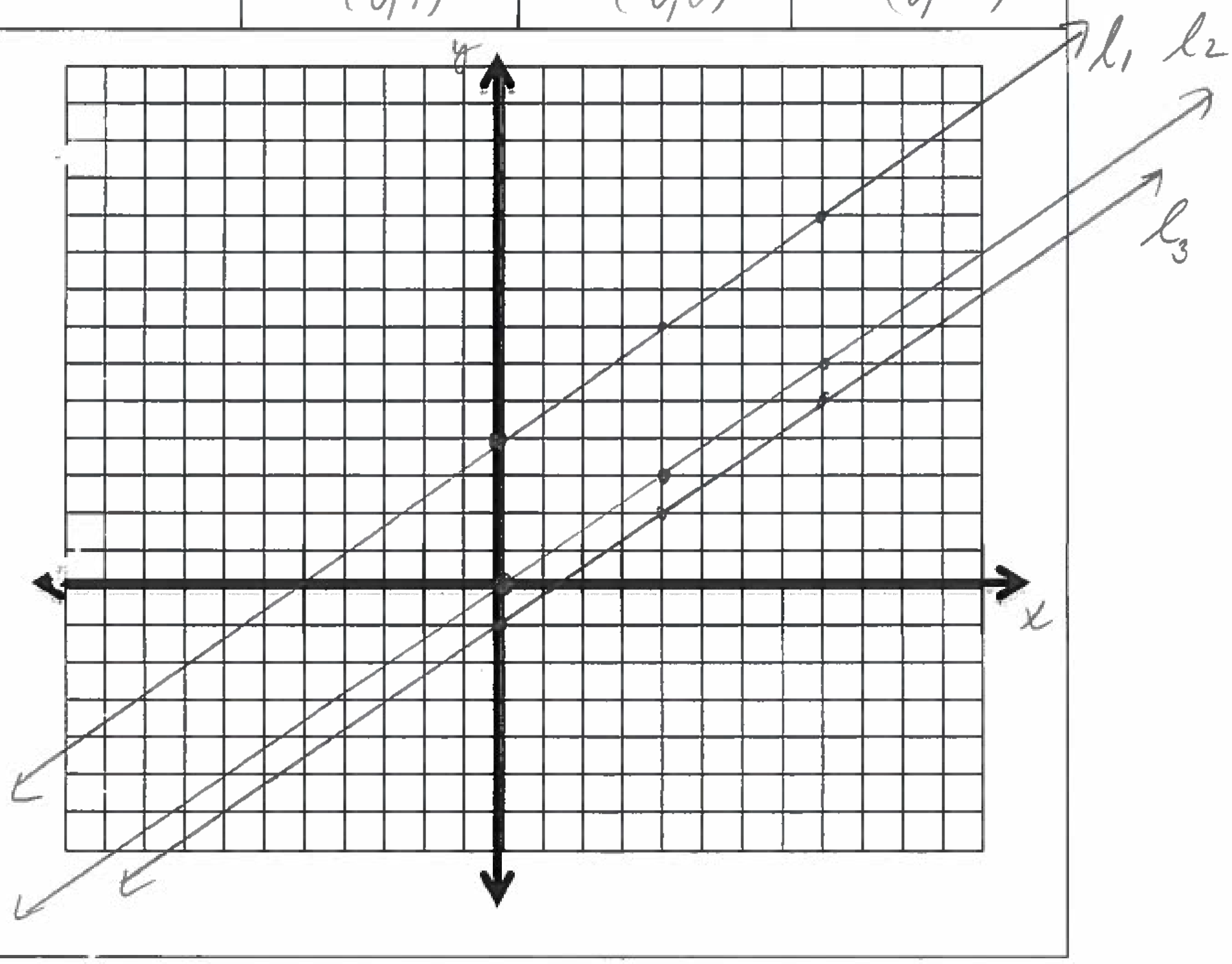
Parallel and Perpendicular Lines

Parallel Lines:

- Lines that never intersect
- Lines that are separated by the same distance from negative to positive infinity
- Lines that have the **same slope** and a different y-intercept OR vertical lines that have a different x-intercept

Example1:

	l_1	l_2	l_3
Equation in Slope Intercept Form	$y = \frac{3}{4}x + 4$	$y = \frac{3}{4}x + 0$	$y = \frac{3}{4}x - 1$
Slope	3/4	3/4	3/4
y-intercept	(0,4)	(0,0)	(0,-1)



$\frac{-1x}{4 \cdot 1} = \frac{-1}{4} \cdot \frac{x}{1} = \frac{-1}{4}x$

general form; change to $y = mx + b$

Example 2: Determine whether lines $y = \frac{-1}{4}x + 3$ and $x + 4y - 16 = 0$ are parallel.

$-x \qquad -x$

$4y - 16 = -x$

$+16 \qquad +16$

$\frac{4y}{4} = \frac{-x}{4} + \frac{16}{4}$

$y = \frac{-1}{4}x + 4$

→ slope : $m = \frac{-1}{4}$

→ y-int. : $(0,4)$

→ slope : $m = \frac{-1}{4}$

→ y-intercept : $(0,3)$

⇒ same slope ✓

⇒ different y-intercepts : $(0,3) \neq (0,4)$ ✓

∴ $y = \frac{-1}{4}x + 3$ is parallel to $x + 4y - 16 = 0$.

Example 3: Determine which of the three given lines is parallel to $y = 3x + 8$

$y + 10 = 3(x + 6)$	$6x - 2y + 16 = 0$	$3x + y = -8$
$y + 10 = 3x + 18$ $-10 \qquad -10$ $y = 3x + 8$ → slope : $m = 3$ → y-int : $(0,8)$ → same slope ✓ → same y-int. X ∴ this line is not parallel.	$-6x \qquad -6x$ $-2y + 16 = -6x$ $-16 \qquad -16$ $\frac{-2y}{-2} = \frac{-6x}{-2} \frac{-16}{-2}$ $y = 3x + 8$ → same slope ✓ → same y-int. X ∴ this line is not parallel to $y = 3x + 8$.	$-3x \qquad -3x$ $y = -3x - 8$ → slope : $m = -3$ → y-int : $(0,8)$ → different slope X → same y-int. : X ∴ this line is not parallel to $y = 3x + 8$.

(not ∥)

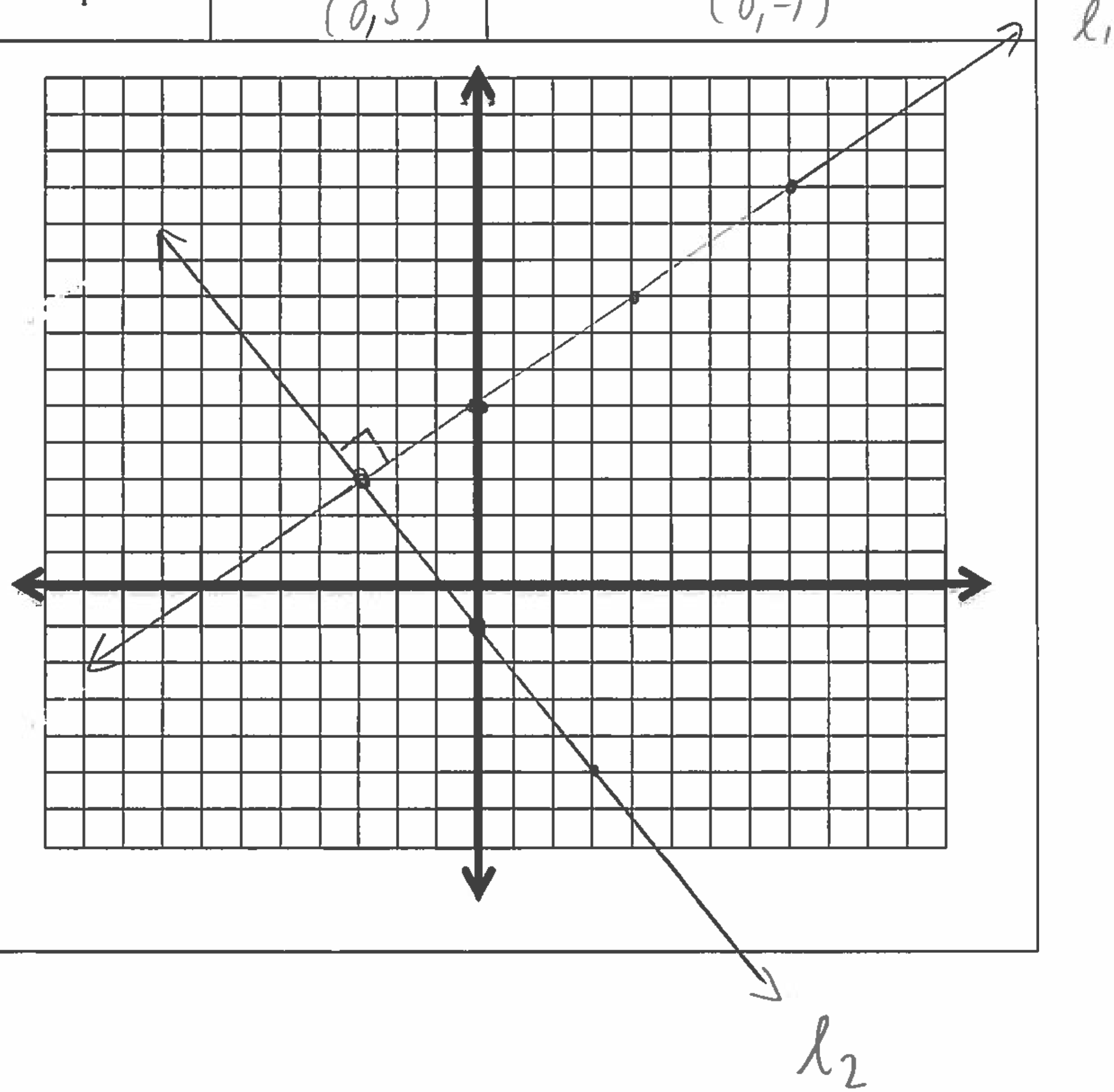
(∦)

Perpendicular Lines:

- Lines that intersect at a 90-degree angle. = Right angle
- Lines with slopes that are a negative reciprocal of each other.
- Perpendicular lines have the same y-intercept if and only if they intersect exactly at the y-intercept.

Example1:

	ℓ_1	ℓ_2
Equation in Slope Intercept Form	$y = \frac{3}{4}x + 5$	$y = \frac{-4}{3}x - 1$
Slope	$3/4$	$-4/3$
y-intercept	$(0,5)$	$(0,-1)$



($\perp$)

Example 2: Determine whether lines $y = \frac{-2}{5}x + 2$ and $5x - 2y - 2 = 0$ are perpendicular.

$-5x \qquad -5x$

$-2y - 2 = -5x$

$+2 \qquad +2$

$\frac{-2y}{-2} = \frac{-5x}{-2} + \frac{2}{-2}$

$y = \frac{5}{2}x - 1$

→ slope: $m = \frac{5}{2}$

→ slope: $m = \frac{-2}{5}$

→ slope $m_\perp = \frac{+5}{2}$

$\therefore$ As $\frac{-2}{5}$ is a negative reciprocal of $\frac{5}{2}$, the lines $y = \frac{-2}{5}x + 2$ and $5x - 2y - 2 = 0$ are perpendicular.

Example 3: Determine which of the three given lines is perpendicular to $y = 3x + 8$ → slope $\boxed{3}$

$y + 1 = \frac{-1}{3}(x + 5)$	$x + 3y - 12 = 0$	$3x + y = 4$
slope: $m = \frac{-1}{3}$ $m_\perp = \frac{+3}{1} = 3$ $\therefore$ This line is perpendicular to $y = 3x + 8$	$-x \qquad -x$ $3y - 12 = -x$ $+12 \qquad +12$ $\frac{3y}{3} = \frac{-x}{3} + \frac{12}{3}$ $y = \frac{-1}{3}x + 4$ $m = \frac{-1}{3}$ $m_\perp = +\frac{3}{1} = 3$ $\therefore$ This line is perpendicular to $y = 3x + 8$	$-3x \qquad -3x$ $y = -3x + 4$ → slope: $m = -3$ $m_\perp = +\frac{1}{3}$ $\frac{1}{3} \neq 3$ $\therefore$ This line is not $\perp$ to $y = 3x + 8$.

Example 4: Determine a negative reciprocal for each number:

$\frac{2}{3}$	$\frac{1}{1}$	$\frac{-5}{1}$	$\frac{4}{7}$	$\frac{10}{1}$	$\frac{-1}{6}$
$-\frac{3}{2}$	$\frac{-1}{1} = -1$	$+\frac{1}{5}$	$-\frac{7}{4}$	$-\frac{1}{10}$	$\frac{6}{-1}$

Linear Relation: Summary

Fill in the blanks:

1. All linear functions have the same domain and range.
(except the horizontal line that is called the Constant Function)
2. All linear relations except vertical lines are functions.
3. To determine whether a relation is a function, one can carry out VLT (*).
4. A horizontal line has a zero slope.
5. A vertical line has an undefined slope. (= infinite)
6. A line with a negative slope is decreasing.
7. A line with a positive slope is increasing.
8. The end behavior of an increasing line is from III quadrant to I quadrant.
9. The end behavior of a decreasing line is from II quadrant to IV quadrant.
10. When two points on the line are known, one can calculate the slope of the line using the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

11. The Slope-Intercept Form of the equation of a line is $y = mx + b$.
12. The Slope-Point Form of the equation of a line is $y - y_1 = m(x - x_1)$.
13. The point where a line intersects the x-axis is called the x-intercept

(*) Vertical line test

and its coordinates are of the form (#, 0).

14. A horizontal line has an equation of the form $y = \#$.

15. In general, horizontal are the only lines that do not have an x-intercept.

16. The x-axis is the only horizontal line that has infinitely many x - intercepts.

17. The point where a line intersects the y-axis is called the y-intercept

and its coordinates are of the form (0, #).

18. A vertical line has an equation of the form $x = \#$.

19. In general, vertical are the only lines that do not have a y-intercept.

20. The y-axis is the only vertical line that has infinitely many y- intercepts.

21. To find the x-intercept, set y = 0 and solve for x. When you find the value of x put it in the form: (#, 0).

22. To find the x-intercept, set x = 0 and solve for y. When you find the value of y put it in the form: (0, #).

23. The General form of the equation of a line is $Ax+By+C=0$.

24. The Standard form of the equation of a line is $Ax + By = C$. *

* Where A, B and C are integers and A is non-negative; also A and B cannot be both zero at the same time.