NOTES

Integral Exponents

Summary of Exponent Rules

for any integers	*m* and *n*	Examples
Exponent of 1	$a^1 = a$	$5^1 = 5$; $(-7)^1 = -7$
Exponent of 0	$a^0 = 1, a \neq 0$,	$5^0 = 1$; $(-7)^0 = 1$
Product Rule	$(a^m)(a^n) = a^{m+n}$	$(2^2)(2^4) = 2^{2+4} = 2^6$ $(4)(16) = 64 = 2^6$
Quotient Rule	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{5^3}{5^2} = 5^{3-2} = 5^1 = 5$
Power Rules	$(a^m)^n = a^{mn}$ $(ab)^m = (a^m)(b^m)$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$	$(2^4)^2 = 2^{4\times 2} = 2^8$ $(3\cdot 7)^2 = 3^2 \cdot 7^2 = 9 \cdot 49 = 441 = 21^2 = 441$ $\left(\frac{x}{7}\right)^2 = \frac{x^2}{7^2} = \frac{x^2}{49} = \frac{1}{49} \cdot x^2$
Negative Exponents	$a^{-n} = \frac{1}{a^n}, a \neq 0$ $\frac{1}{a^{-n}} = a^n$ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ $a \neq 0, b \neq 0$	$5^{-1} = \frac{1}{5^1} = \frac{1}{5}$ $\frac{1}{3^{-4}} = 3^{+4} = 81$ $\left(\frac{x}{3}\right)^{-5} = \left(\frac{3}{x}\right)^5$
Rational Exponents	$\sqrt[n]{a} = a^{\frac{1}{n}}$ $\sqrt[n]{a^m} = a^{\frac{m}{n}}$	$\sqrt{x} = x^{\frac{1}{2}}$ $\sqrt[3]{x^6} = x^{\frac{6}{3}} = x^2$

new ! • extra !!

↑ Flower-Power Rule

Simplifying Integral Exponents

1. Simplify.

a) $(5^8)(5^{-3}) = 5^{8+(-3)} = 5^{8-3} = \boxed{5^5}$

Simply using positive exponents.

$(5^8)(5^{-3}) = \frac{5^8}{1} \cdot \frac{1}{5^3} = \frac{5^8}{5^3} = 5^{8-3} = \boxed{5^5}$

b) $(0.8^{-2})(0.8)^{-4}$

$= 0.8^{-2+(-4)} = \boxed{0.8^{-6}}$

$(0.8^{-2})(0.8)^{-4} = \left(\frac{8}{10}\right)^{-2}\left(\frac{8}{10}\right)^{-4}$

$= \left(\frac{5}{4}\right)^2\left(\frac{5}{4}\right)^4 = \left(\frac{5}{4}\right)^{2+4}$

$= \left(\frac{5}{4}\right)^6 = \frac{5^6}{4^6} = \frac{15625}{4096}$

c) $\frac{(2x)^5}{(2x)^{-3}} = (2x)^{5-(-3)}$

$= (2x)^8 = 2^8 x^8$

$= \boxed{256x^8}$

$\frac{(2x)^5}{(2x)^{-3}}$

$\frac{(2x)^5}{1} \cdot \frac{1}{(2x)^{-3}} = \frac{(2x)^5}{1} \cdot \frac{(2x)^3}{1} = (2x)^{5+3}$

$= \boxed{(2x)^8}$

$\boxed{256x^8} = 2^8 x^8$

Your Turn:

a) $(2^{-3})(2)^5$

$2^{(-3)+5}$

$\boxed{2^2 = 4}$

b) $\frac{7^{-5}}{7^3} = 7^{-5-3}$

$= 7^{-8}$

$= \boxed{\frac{1}{7^8}}$

c) $\frac{(-3.5)^4}{(-3.5)^{-3}} = (-3.5)^{4-(-3)}$

$= (-3.5)^7$

$= \left[\frac{-35}{10}\right]^7$

$= \left[\frac{-7}{5}\right]^7 = \frac{(-7)^7}{5}$

2. Simplify to a power with a single, positive exponent; evaluate where possible.

a) $(4^3)^{-2} = 4^{(3)(-2)} = 4^{-6} = \frac{1}{4^6}$

OR $\doteq 0.0002$ $= \boxed{\frac{1}{4096}}$

b) $[(a^{-2})(a^0)]^{-1} = [a^{(-2)+0}]^{-1} = [a^{-2}]^{-1}$

where $a \neq 0$ $= a^{(-2)(-1)} = \boxed{a^2}$

c) $\left(\frac{2^4}{2^6}\right)^{-3} = \left(\frac{2^6}{2^4}\right)^3$

$= (2^{6-4})^3$

$= (2^2)^3 \rightarrow$ OR $= 2^{(2)(3)}$

$= 4^3$ $= 2^6$

$= 64$ $= 64$

d) $\left[\left(\frac{3}{4}\right)^{-2}\left(\frac{3}{4}\right)^4\right]^{-2} = \left[\left(\frac{3}{4}\right)^{(-2)+4}\right]^{-2} = \left[\left(\frac{3}{4}\right)^2\right]^{-2}$

$= \left(\frac{3}{4}\right)^{(2)(-2)} = \left(\frac{3}{4}\right)^{-4} = \left(\frac{4}{3}\right)^4 =$

$= \frac{4^4}{3^4} = \frac{256}{81} \doteq 3.161$

Your Turn:

a) $\left[(0.6^3)(0.6)^{-3}\right]^{-5}$

$= \left[(0.6)^{3+(-3)}\right]^{-5}$

$= [0.6^0]^{-5}$

$= [1]^{-5}$

$= \frac{1}{1^5} = \frac{1}{1} = \boxed{1}$

b) $\left(\frac{x^6}{x^4}\right)^{-2} = \left(\frac{x^4}{x^6}\right)^2$

$= (x^{4-6})^2$

$= (x^{-2})^2$

$= x^{(-2)(2)}$

$= x^{-4}$

$= \boxed{\frac{1}{x^4}}$

c) $\left[\frac{(y^2)^0}{y^3}\right]^{-3} = \left[\frac{y^3}{(y^2)^0}\right]^{+3} = \left[\frac{y^3}{y^{(2)(0)}}\right]^3$

$= \left[\frac{y^3}{y^0}\right]^3 = [y^{3-0}]^3$

$= [y^3]^3 = y^{(3)(3)} = \boxed{y^9}$

3. A culture bacteria in a lab contains 2000 bacterium cells. The number of cells doubles every day. This relationship can be modeled by the equation $N = 2000(2)^t$, where **N** is the estimated number of bacteria cells and ***t*** is the time in days. How many cells were present after two days?

$N = 2000(2)^2$
$N = 8000$

There will be 8000 bacterium cells after 2 days of growth.

How many cells were present after one week? How many cells were present 2 days ago?

7 days

$N = 2000(2)^7$
$N = 256\,000$

-2

$N = 2000(2)^{-2}$
$N = 500$

4. A mountain pine beetle population can triple every year. If the population in Jasper National Park is 10,000 the formula for the population would be $P=10{,}000(3)^n$, n being the # of years.

How many beetles will there be 2 years from now?

$P = 10000(3)^2$
$P = 90000$

How many beetles were there 4 years ago?

$P = 10000(3)^{-4}$
$P = 123.$

5. There are ~ 117 billion grasshoppers in an area of 39,000 km^2. How many are there per square kilometer? Use exponents to solve.

$$\frac{117 \times 10^9}{3.9 \times 10^4} = \frac{117}{3.9} \times 10^{9-4} = 30 \times 10^5$$

$$= 3000000$$

$$= 3.0 \times 10^6 \text{ grasshoppers}$$