

FMPC10

ARITHMETIC SEQUENCES – introduction

- In mathematics, a sequence is a list of numbers or algebraic expressions separated by commas. For a list to be called a sequence, there must be a pattern between each pair of neighbouring numbers or expressions.
- When the pattern between each pair of numbers or expressions is a result of addition (or subtraction) we call this sequence an arithmetic sequence.

Examples of arithmetic sequences:

- $-4, -2, 0, 2, 4, 6, 8, 10, \dots$ \leftarrow the pattern continues.
 $d = 2$
- $35, 30, 25, 20, 15, 10, 5, 0, \dots$
 $d = -5$
- $x + 3, 2x + 1, 3x - 1, 4x - 3, \dots$
 $d = x - 2$

Examples of sequences that are not arithmetic:

- $7, 21, 63, 189, \dots$ \rightarrow geometric sequence (b/c pattern is thru multiplication)
- $-2, 1, -0.5, 0.25, -0.125, \dots$ \rightarrow geometric (not arithmetic)
- $x + 3, 2x + 6, 4x + 12, 8x + 24, \dots$ geometric.
- $0, 1, 1, 2, 3, 5, 8, 13, 21, \dots$ Fibonacci Sequence

Examples of lists that are not sequences:

- $7, 10, 14, 19, \dots$ (there is a pattern, but not a sequence)
- $3, 9, 27, 81, 82, \dots$ (pattern doesn't continue)
- $2, 24, 12, 144, 132, \dots$ (pattern but not a sequence)



- Every number or expression in the arithmetic sequence is called **a term**.
- Every term has a subscript that is called an **index** (or sometimes a counter) and this **index identifies the placement of the term in the sequence**. This subscript works like an address for each term. The symbol for any subscript is a lower case "n".
- A symbol used for any term in the sequence is either a lower case "t" or a lower case "a".
- The number or expression that two neighbouring terms differ by is called the **common difference**. The symbol for a common difference is a lower case "d".

Example:

- $35, 30, 25, 20, 15, 10, 5, 0, \dots$

The common difference is: $d = -5$

The first term is: $t_1 = 35$ OR $a_1 = 35$

The fourth term is: $t_4 = 20$ OR $a_4 = 20$

Every term in an arithmetic sequence is given by an equation that can be written in several ways. The two most common ways are:

$t_n = t_1 + (n - 1)d$	$a_n = a_1 + (n - 1)d$
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In general, the common difference can be found by taking any term in the list and subtracting the nearest neighbour to the left.

any term except the first one.

$$d = t_n - t_{n-1}$$

term on the left of the chosen one.

- Be careful, you cannot assume that a list of numbers or expressions is an arithmetic sequence unless you are told so or you **check every pair of neighbouring numbers/expressions to see if all of them have the same difference**. If there is a single pair of neighbouring numbers or expressions that do not have the same difference, the list is not a sequence.
- The three dots at the end of the list that follow the last comma are an important in mathematics, they communicate that the pattern observed at the beginning of the list continues into infinity.

$[\dots]$ is a math symbol, which means the pattern continues forever.

$$t_n = t_1 + (n-1)d$$

- Arithmetic Sequences can be graphed. As "n" stands for counting numbers (or integers), the graph is formed by isolated points that lay along a line.

$$t_2 = 3 - 2 = 1$$

$$t_3 = 1 - 2 = -1$$

Example: $t_n = 3 + (n-1)(-2)$

$$t_1 = 3$$

$$d = -2$$



