

Continuity at a Point

Interior point: A function $y=f(x)$ is continuous at an interior point c of its domain if

$$\lim_{x \rightarrow c} f(x) = f(c)$$

End point: A function $y=f(x)$ is continuous at a left endpoint a or is continuous at a right endpoint b of its domain if

$$\lim_{x \rightarrow a^+} f(x) = f(a) \text{ and } \lim_{x \rightarrow b^-} f(x) = f(b), \text{ respectively}$$

- Recall: A continuous function is a function that is continuous at every point of its domain. A continuous function is not always continuous on every interval.

Properties of continuous functions:

- If $f(x)$ and $g(x)$ are both continuous at $x=c$, then their sum, difference, product, constant multiple and quotient (provided that $g(c)$ is not zero) are all continuous at $x=c$.
- If $g(x)$ is continuous at $x=c$ and $f(x)$ is continuous at $g(c)$, then the composite function $f(g(x))$ is continuous at $x=c$.

To determine whether a function is continuous at $x=c$ the following must be true:

1. The given function is defined at $x=c$.
2. The two-sided limit of the function exists as x approaches c .
3. The value of the function at $x=c$ is the same as the two-sided limit as x approaches c .

The Intermediate Value Theorem (IVT) for Continuous Functions

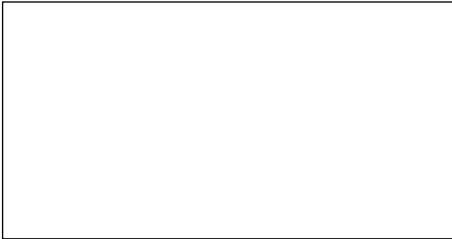
A function $y=f(x)$ that is continuous on a closed interval $[a,b]$ takes on every value between $f(a)$ and $f(b)$. In other words if y_0 is between $f(a)$ and $f(b)$, then $y_0 = f(c)$ for some c in $[a,b]$.

Floor Function

Ceiling Function

Average Rate of Change

- Given by the slope of a secant line.
- The average rate of change of $f(x)$ over the interval $[a,b]$ is given by:



Instantaneous Rate of Change

- Given by the slope of a tangent line.
- The instantaneous rate of change of $f(x)$ at a point $P(a, f(a))$ is the slope of the curve at this point and it is identical to the slope of the tangent line to $f(x)$ at point P .
- This slope m is given by:

Provided that the limit exists.