

Factoring: All Techniques Combined (Hard)

Factor each.

$$\begin{aligned}
 1) \quad x^3 - 5x^2 - x + 5 \\
 &= x^2(x-5) - 1(x-5) \\
 &= (x-5)(x^2-1) \\
 &= \underline{(x-5)(x-1)(x+1)}
 \end{aligned}$$

$$\begin{aligned}
 3) \quad x^6 - 26x^3 - 27 \quad \text{let } A = x^3 \\
 A^2 - 26A - 27 \\
 = (A-27)(A+1) \\
 = (x^3-27)(x^3+1) \leftarrow \text{cubes} \\
 = \underline{(x-3)(x^2+3x+9)(x+1)(x^2-x+1)}
 \end{aligned}$$

$$\begin{aligned}
 5) \quad x^4 - 13x^2 + 40 \\
 \text{let } A = x^2 \\
 A^2 - 13A + 40 \\
 = (A-8)(A-5) \\
 = \underline{(x^2-8)(x^2-5)}
 \end{aligned}$$

$$\begin{aligned}
 7) \quad x^6 - 4x^2 = x^2(x^4-4) \\
 = \underline{x^2(x^2-2)(x^2+2)}
 \end{aligned}$$

OR

$$= x^2(x-\sqrt{2})(x+\sqrt{2})(x^2+2)$$

$$\begin{aligned}
 2) \quad x^4 - 2x^2 - 15 \quad \text{let } A = x^2 \\
 = \underline{A^2 - 2A - 15} \\
 = (x^2-5)(x^2+3) \\
 \text{Substitute back } A = x^2 \\
 = \underline{(A-5)(A+3)}
 \end{aligned}$$

$$\begin{aligned}
 4) \quad x^6 + 2x^4 - 16x^2 - 32 \\
 = x^4(x^2+2) - 16(x^2+2) \\
 = (x^2+2)(x^4-16) \\
 = (x^2+2)(x^2-4)(x^2+4) \\
 = \underline{(x^2+2)(x^2+4)(x-2)(x+2)}
 \end{aligned}$$

$$\begin{aligned}
 6) \quad x^9 - x^6 - x^3 + 1 \\
 = x^6(x^3-1) - 1(x^3-1) \\
 = (x^6-1)(x^3-1) \\
 = (x^3-1)(x^3+1)(x^3-1) = \underbrace{(x^3-1)^2(x^3+1)}_{\text{cubes}} \\
 = \underline{(x-1)^2(x^2+x+1)^2(x+1)(x^2-x+1)}
 \end{aligned}$$

$$\begin{aligned}
 8) \quad x^4 + 14x^2 + 45 \\
 A = x^2 \rightarrow A^2 + 14A + 45 \\
 = (A+9)(A+5) \\
 = \underline{(x^2+9)(x^2+5)}
 \end{aligned}$$

$$9) 2x^4 + x^2 - 6 \quad A = x^2$$

$$2A^2 + A - 6$$

$$2A^2 + 4A - 3A - 6$$

$$2A(A+2) - 3(A+2)$$

$$(x^2+2)(2x^2-3)$$

$$11) 4x^3 - x^2 - 4x + 1$$

$$x^2(4x-1) - 1(4x-1)$$

$$(x^2-1)(4x-1)$$

$$(x-1)(x+1)(4x-1)$$

$$13) 5x^2 + 24x - 5$$

$$5x^2 - x + 25x - 5$$

$$x(5x-1) + 5(5x-1)$$

$$(x+5)(5x-1)$$

$$15) 4x^2 + 4x - 15$$

$$4x^2 - 6x + 10x - 15$$

$$2x(2x-3) + 5(2x-3)$$

$$(2x+5)(2x-3)$$

$$17) \frac{-64x^3 + 125}{-1 \quad -1 \quad -1} = 0$$

$$64x^3 - 125 = 0$$

$$(4x-5)(16x^2 + 20x + 25) = 0$$

$$(4x-5)(16x^2 + 20x + 25) = 0$$

$$10) 2x^2 - 13x + 20$$

$$= 2x^2 - 8x - 5x + 20$$

$$= 2x(x-4) - 5(x-4)$$

$$= (2x-5)(x-4)$$

$$12) 4x^8 - 61x^4 + 225$$

$$= 4x^8 - 25x^4 - 36x^4 + 225$$

$$= x^4(4x^4 - 25) - 9(4x^4 - 25)$$

$$= (x^4 - 9)(4x^4 - 25)$$

$$= (x^2 - 3)(x^2 + 3)(2x^2 - 5)(2x^2 + 5)$$

$$14) 5x^2 + 29x + 20$$

$$= 5x^2 + 4x + 25x + 20$$

$$= x(5x+4) + 5(5x+4)$$

$$= (x+5)(5x+4)$$

$$16) 10x^3 - 8x^2 + 25x - 20$$

$$= 2x^2(5x-4) + 5(x-4)$$

$$= (2x^2+5)(5x-4)$$

$$18) 8x^4 + 10x^2 - 3$$

$$= 8x^4 - 2x^2 + 12x^2 - 3$$

$$= 2x^2(4x^2-1) + 3(4x^2-1)$$

$$= (2x^2+3)(4x^2-1)$$

$$= (2x^2+3)(2x-1)(2x+1)$$