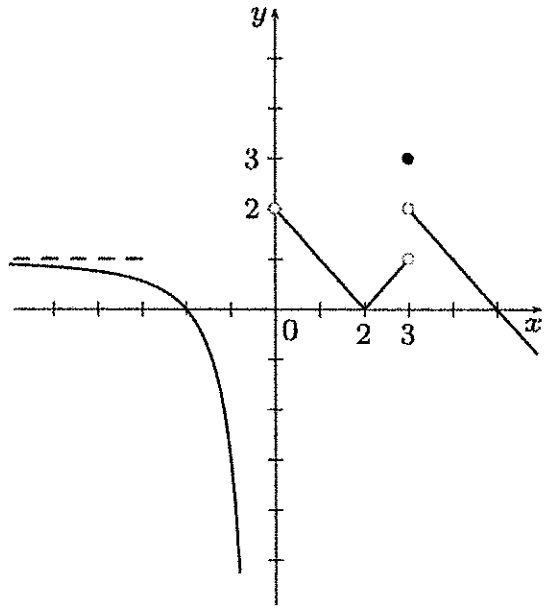
201-103-RE - Calculus 1

WORKSHEET: LIMITS

1. Use the graph of the function $f(x)$ to answer each question.
   Use ∞, $-\infty$ or DNE where appropriate.



(a) $f(0) =$

(b) $f(2) =$

(c) $f(3) =$

(d) $\lim_{x \to 0^-} f(x) =$

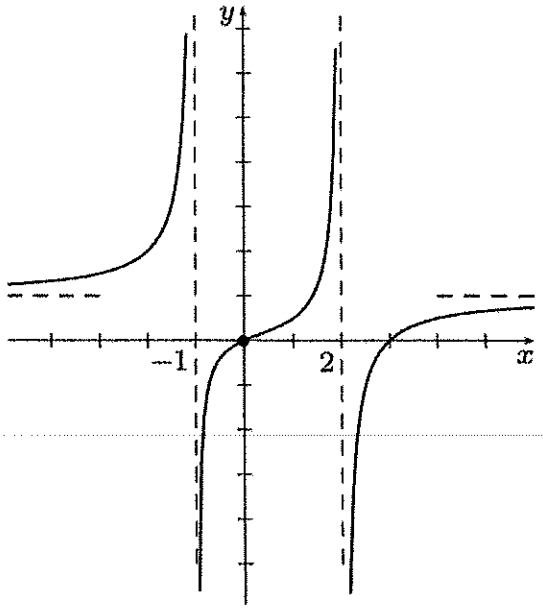
(e) $\lim_{x \to 0} f(x) =$

(f) $\lim_{x \to 3^+} f(x) =$

(g) $\lim_{x \to 3} f(x) =$

(h) $\lim_{x \to -\infty} f(x) =$

2. Use the graph of the function $f(x)$ to answer each question.
   Use ∞, $-\infty$ or DNE where appropriate.



(a) $f(0) =$

(b) $f(2) =$

(c) $f(3) =$

(d) $\lim_{x \to -1} f(x) =$

(e) $\lim_{x \to 0} f(x) =$

(f) $\lim_{x \to 2^+} f(x) =$

(g) $\lim_{x \to \infty} f(x) =$

3. Evaluate each limit using algebraic techniques.
Use ∞, $-\infty$ or DNE where appropriate.

(a) $\lim_{x\to 0} \dfrac{x^2-25}{x^2-4x-5}$

(b) $\lim_{x\to 5} \dfrac{x^2-25}{x^2-4x-5}$

(c) $\lim_{x\to 1} \dfrac{7x^2-4x-3}{3x^2-4x+1}$

(d) $\lim_{x\to -2} \dfrac{x^4+5x^3+6x^2}{x^2(x+1)-4(x+1)}$

(e) $\lim_{x\to -3} |x+1| + \dfrac{3}{x}$

(f) $\lim_{x\to 3} \dfrac{\sqrt{x+1}-2}{x^2-9}$

(g) $\lim_{x\to 3} \dfrac{\sqrt{x^2+7}-3}{x+3}$

(h) $\lim_{x\to 2} \dfrac{x^2+2x-8}{\sqrt{x^2+5}-(x+1)}$

(i) $\lim_{y\to 5} \left(\dfrac{2y^2+2y+4}{6y-3}\right)^{1/3}$

(j) $\lim_{x\to 0} \sqrt[4]{2\cos(x)-5}$

(k) $\lim_{x\to 0} \dfrac{\dfrac{1}{3+x}-\dfrac{1}{3-x}}{x}$

(l) $\lim_{x\to -6} \dfrac{\dfrac{2x+8}{x^2-12}-\dfrac{1}{x}}{x+6}$

(m) $\lim_{x\to \infty} \sqrt{x^2-2}-\sqrt{x^2+1}$

(n) $\lim_{x\to -\infty} \sqrt{x-2}-\sqrt{x}$

(o) $\lim_{x\to 7} \sqrt[6]{2x-14}$

(p) $\lim_{x\to 1^-} \sqrt{3-3x}$

(q) $\lim_{x\to \infty} \dfrac{x^4-10}{4x^3+x}$

(r) $\lim_{x\to -\infty} \sqrt[3]{\dfrac{x-3}{5-x}}$

(s) $\lim_{x\to \infty} \dfrac{3x^3+x^2-2}{x^2+x-2x^3+1}$

(t) $\lim_{x\to \infty} \dfrac{x+5}{2x^2+1}$

(u) $\lim_{x\to -\infty} \cos\left(\dfrac{x^5+1}{x^6+x^5+100}\right)$

(v) $\lim_{x\to 2} \dfrac{2x}{x^2-4}$

(w) $\lim_{x\to -1} \dfrac{3x}{x^2+2x+1}$

(x) $\lim_{x\to -1} \dfrac{x^2-25}{x^2-4x-5}$

(y) $\lim_{x\to 3} \dfrac{\sqrt{x^2-5}+2}{x-3}$

(z) $\lim_{x\to 0} \dfrac{2^x+\sin(x)}{x^4}$

(A) $\lim_{x\to 1^-} \dfrac{1}{x-1} + e^{x^2}$

(B) $\lim_{x\to \infty} 2x^2-3x$

(C) $\lim_{x\to 0} \dfrac{\sqrt{x+2}-\sqrt{2-x}}{x}$

(D) $\lim_{x\to 0^+} \dfrac{e^x}{1+\ln(x)}$

(E) $\lim_{x\to \infty} \sqrt{x^2+1}-2x$

(F) $\lim_{x\to 1} \dfrac{\sqrt[3]{x}-1}{\sqrt{x}-1}$

4. Find the following limits involving absolute values.

(a) $\lim_{x\to 1} \dfrac{x^2-1}{|x-1|}$ (b) $\lim_{x\to -2} \dfrac{1}{|x+2|} + x^2$ (c) $\lim_{x\to 3^-} \dfrac{x^2|x-3|}{x-3}$

5. Find the value of the parameter k to make the following limit exist and be finite. What is then the value of the limit?

$$\lim_{x\to 5} \frac{x^2+kx-20}{x-5}$$

6. Answer the following questions for the piecewise defined function $f(x)$ described on the right hand side.

(a) $f(1) =$

(b) $\lim_{x\to 0} f(x) =$

(c) $\lim_{x\to 1} f(x) =$

$$f(x) = \begin{cases} \sin(\pi x) & \text{for } x < 1, \\ 2^{x^2} & \text{for } x > 1. \end{cases}$$

7. Answer the following questions for the piecewise defined function $f(t)$ described on the right hand side.

(a) $f(-3/2) =$

(b) $f(2) =$

(c) $f(3/2) =$

(d) $\lim_{t\to -2} f(t) =$

(e) $\lim_{t\to -1^+} f(t) =$

(f) $\lim_{t\to 2} f(t) =$

(g) $\lim_{t\to 0} f(t) =$

$$f(t) = \begin{cases} t^2 & \text{for } t < -2 \\ \dfrac{t+6}{t^2-t} & \text{for } -1 < t < 2 \\ 3t-2 & \text{for } t \geq 2 \end{cases}$$

ANSWERS:

1. (a) DNE (b) 0 (c) 3 (d) $-\infty$ (e) DNE (f) 2 (g) DNE (h) 1
2. (a) 0 (b) DNE (c) 0 (d) DNE (e) 0 (f) $-\infty$ (g) 1
3.
   (a) 5
   (b) $\frac{5}{3}$
   (c) 5
   (d) 1
   (e) 1
   (f) $\frac{1}{24}$
   (g) $\frac{1}{6}$
   (h) -18
   (i) $\frac{4}{3}$
   (j) DNE
   (k) $-\frac{2}{9}$
   (l) $\frac{1}{36}$
   (m) 0
   (n) DNE
   (o) DNE
   (p) 0
   (q) ∞
   (r) -1
   (s) $-\frac{3}{2}$
   (t) 0
   (u) 1
   (v) DNE
   (w) $-\infty$
   (x) DNE
   (y) DNE
   (z) ∞
   (A) $-\infty$
   (B) ∞
   (C) $\frac{1}{\sqrt{2}}$
   (D) 0
   (E) $-\infty$
   (F) $\frac{2}{3}$
4. (a) DNE (b) ∞ (c) -9
5. $k = -1$, limit is then equal to 9
6. (a) DNE (b) 0 (c) DNE
7. (a) DNE (b) 4 (c) 10 (d) DNE (e) $\frac{5}{2}$ (f) 4 (g) DNE
8. (a) 0 (b) 0 (c) $\frac{5}{3}$