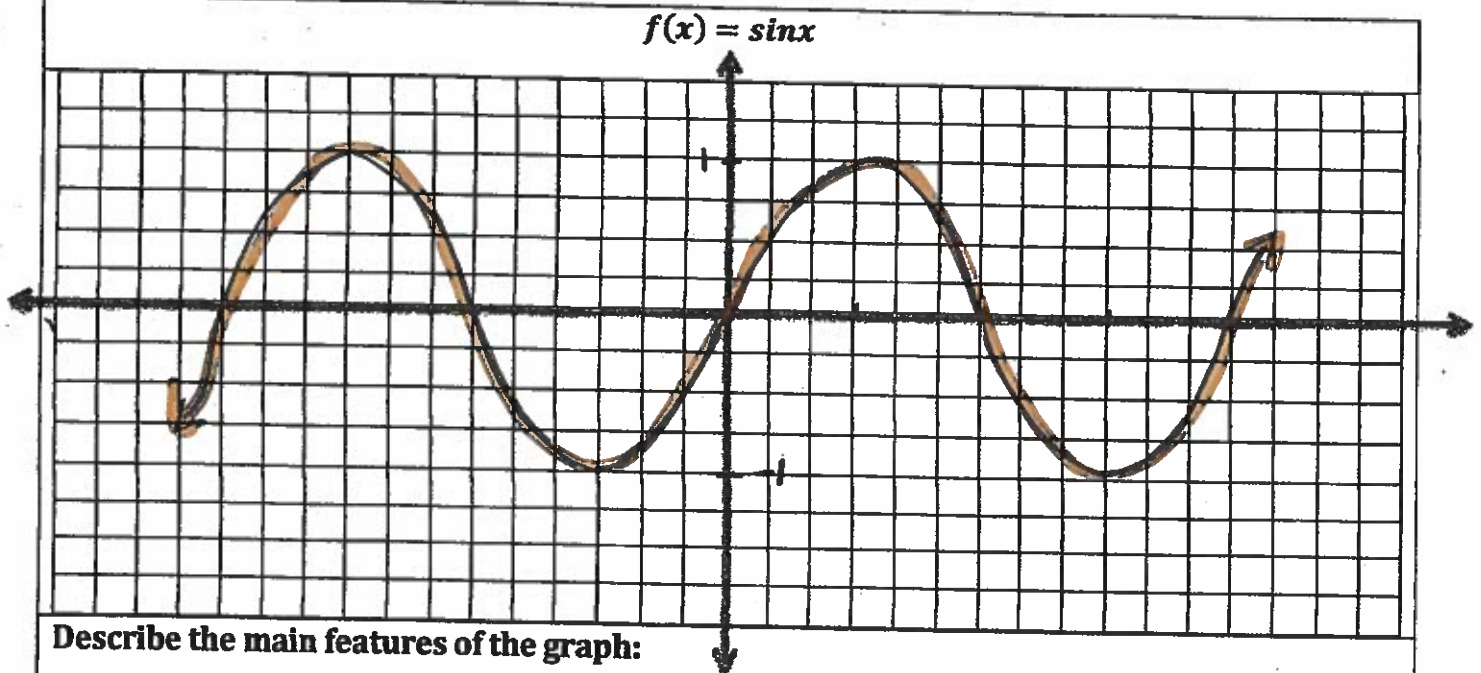


# TRIGONOMETRIC FUNCTIONS

## 1. Basic Trigonometric Functions



Describe the main features of the graph:

General coordinates of x-intercepts:  $(n\pi, 0)$  where  $n \in \mathbb{Z}$

y-intercept:  $(0, 0)$

Maximum value:  $y = 1$

Minimum value:  $y = -1$

Amplitude:  $a = 1$

Period:  $2\pi$

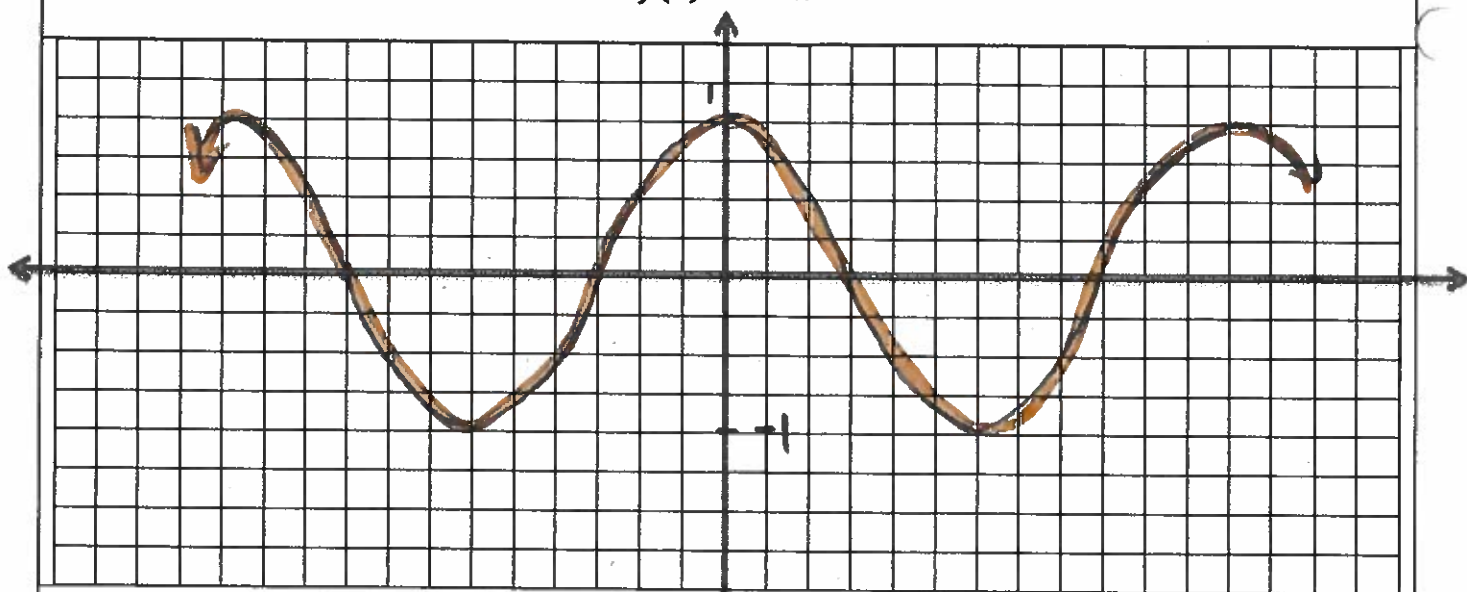
Domain:

$$D: \{x \mid x \in \mathbb{R}\}$$

Range:

$$R: \{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$$

$$f(x) = \cos x$$



Describe the main features of the graph:

General coordinates of x-intercepts:  $\left( (2n-1)\frac{\pi}{2}, 0 \right)$  where  $n \in \mathbb{Z}$

y-intercept:  $(0, 1)$

Maximum value:  $y = 1$

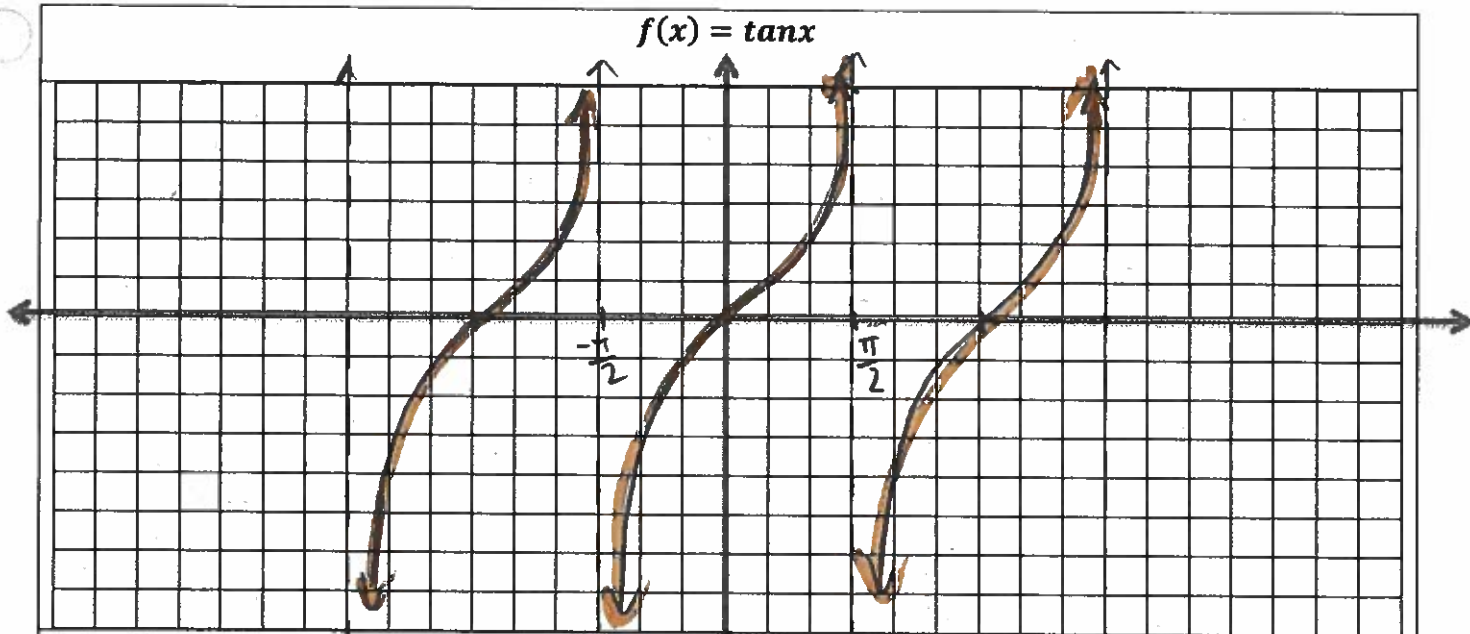
Minimum value:  $y = -1$

Amplitude:  $a = 1$

Period:  $2\pi$

Domain:  $\mathbb{D} : \{x \mid x \in \mathbb{R}\}$

Range:  $\mathbb{R} : \{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$



Describe the main features of the graph:

General coordinates of x-intercepts:  $(n\pi, 0) \quad n \in \mathbb{Z}$

y-intercept:  $(0, 0)$

Maximum value: none

Minimum value: none

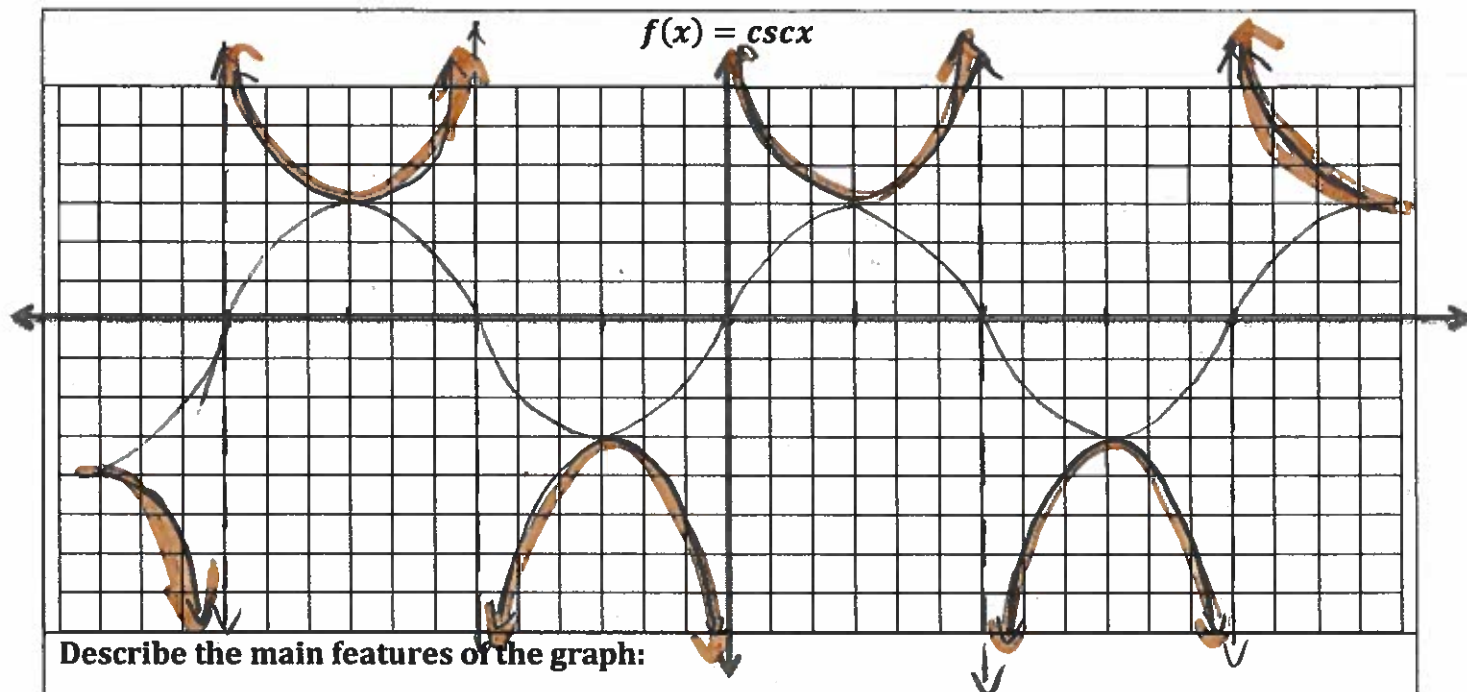
General equation of vertical asymptotes:  $VA: x = (2n-1)\frac{\pi}{2}, \text{ where } n \in \mathbb{Z}$

Period:  $\pi$

Domain:  $D: \{x \mid x \neq (2n-1)\frac{\pi}{2}, n \in \mathbb{Z}, x \in \mathbb{R}\}$

Range:  $R: \{y \mid y \in \mathbb{R}\}$

## 2. Reciprocal Trigonometric Functions



Describe the main features of the graph:

General coordinates of x-intercepts: none

y-intercept: none

Maximum value: none

Minimum value: none

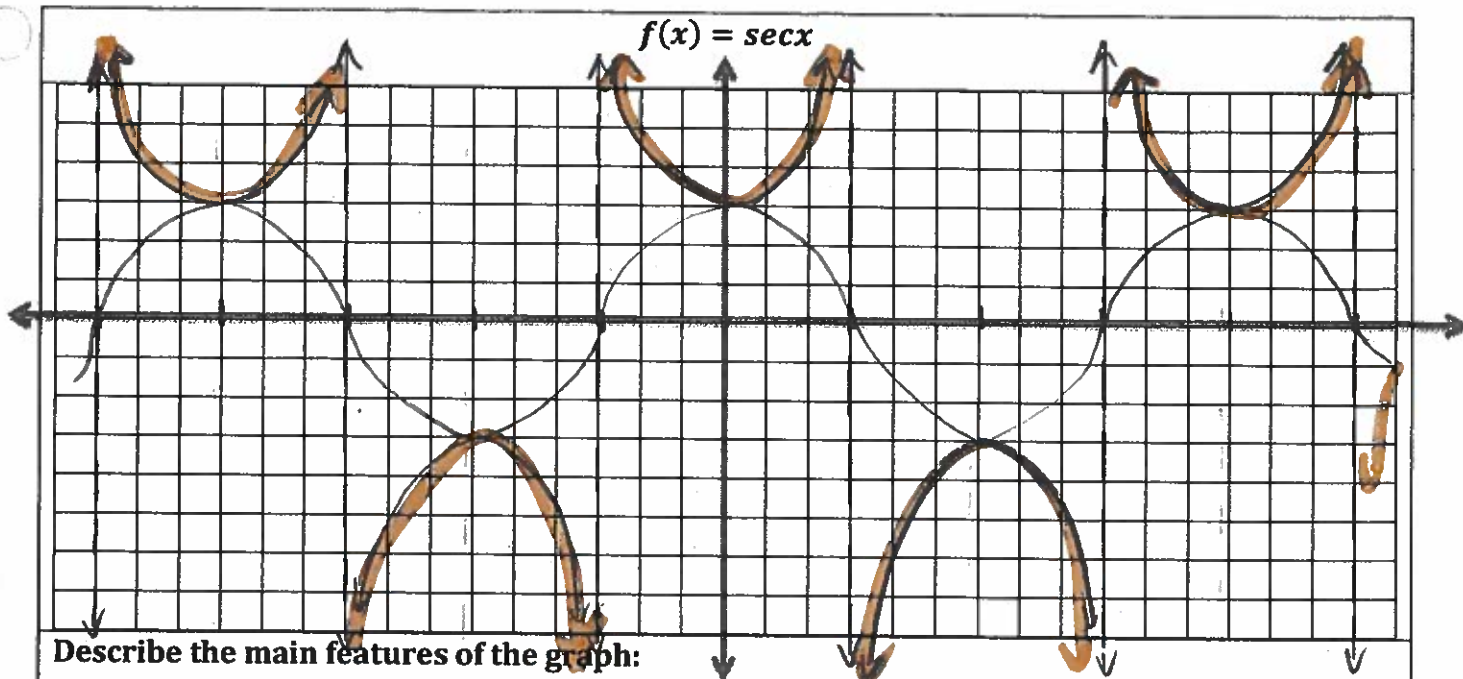
$$\csc x = \frac{1}{\sin x}$$

General equation of vertical asymptotes: VA:  $x = n\pi$ , where  $n \in \mathbb{Z}$

Period:  $2\pi$

Domain:  $D: \{x \mid x \neq n\pi, n \in \mathbb{Z}, x \in \mathbb{R}\}$

Range:  $R: \{y \mid y \leq -1 \text{ and } y \geq 1, y \in \mathbb{R}\}$



General coordinates of x-intercepts: *none*

y-intercept:  $(0, 1)$

Maximum value: *none*

Minimum value: *none*

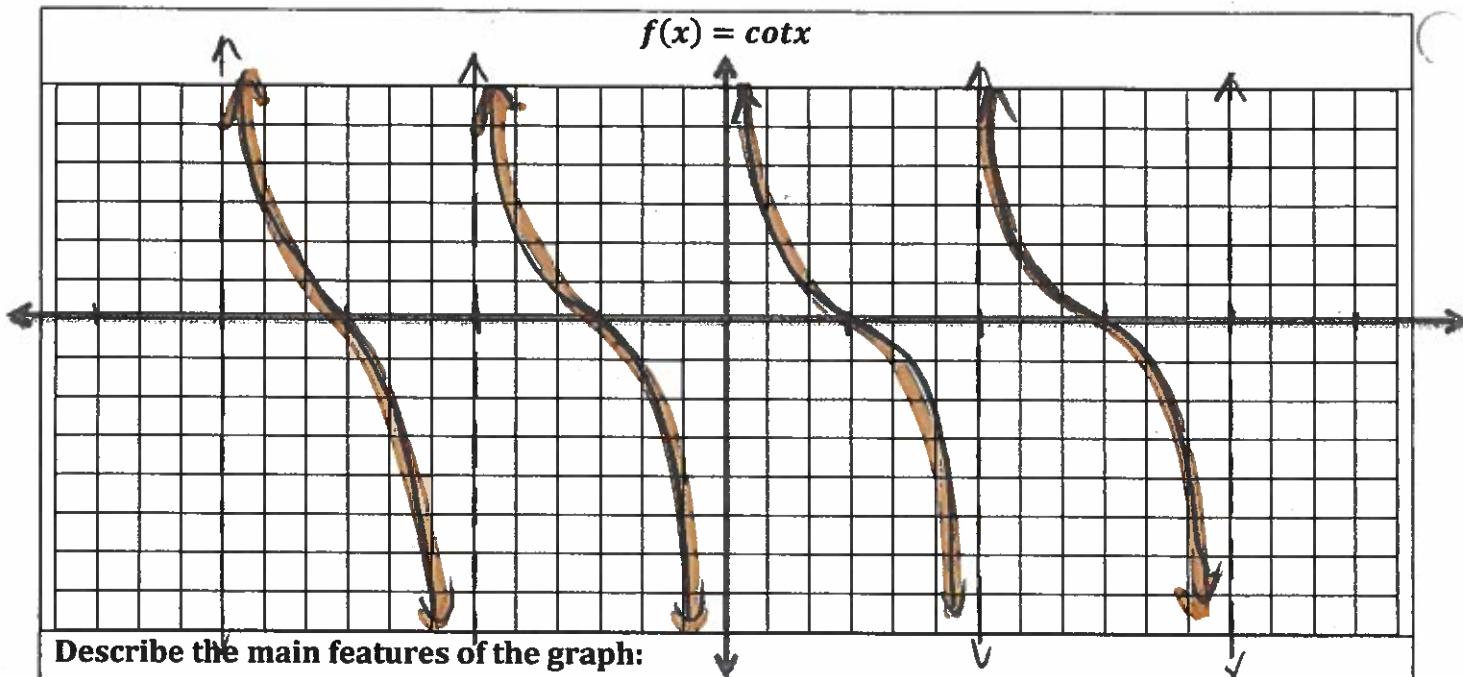
$$\sec x = \frac{1}{\cos x}$$

General equation of vertical asymptotes: VA:  $x = (2n-1)\frac{\pi}{2}$ , where  $n \in \mathbb{Z}$

Period:  $2\pi$

Domain:  $D: \{x \mid x \neq (2n-1)\frac{\pi}{2}, n \in \mathbb{Z}, x \in \mathbb{R}\}$

Range:  $R: \{y \mid y \geq 1 \text{ and } y \leq -1, y \in \mathbb{R}\}$



**Describe the main features of the graph:**

General coordinates of x-intercepts:  $( (2n-1)\frac{\pi}{2}, 0 )$

y-intercept: none

Maximum value: none

Minimum value: none

$$\cot x = \frac{1}{\tan x}$$

General equation of vertical asymptotes:

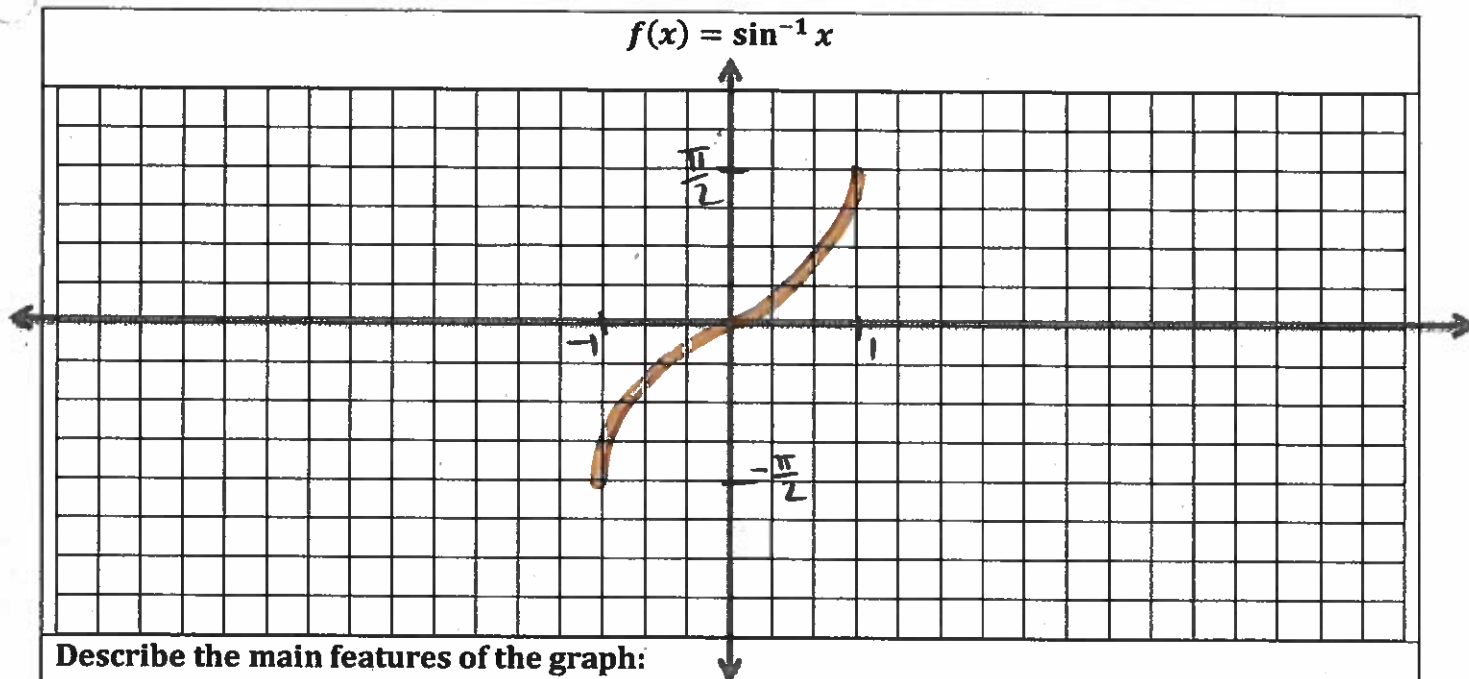
$$VA: x = n\pi, \text{ where } n \in \mathbb{Z}$$

Period:  $\pi$

Domain:  $D: \{ x \mid x \neq n\pi, n \in \mathbb{Z}, x \in \mathbb{R} \}$

Range:  $R: \{ y \mid y \in \mathbb{R} \}$

### 3. Inverse Trigonometric Functions



Describe the main features of the graph:

~~General coordinates of x-intercept:~~  $(0,0)$

y-intercept:  $(0,0)$

Maximum value:  $y = \frac{\pi}{2}$

Minimum value:  $y = -\frac{\pi}{2}$

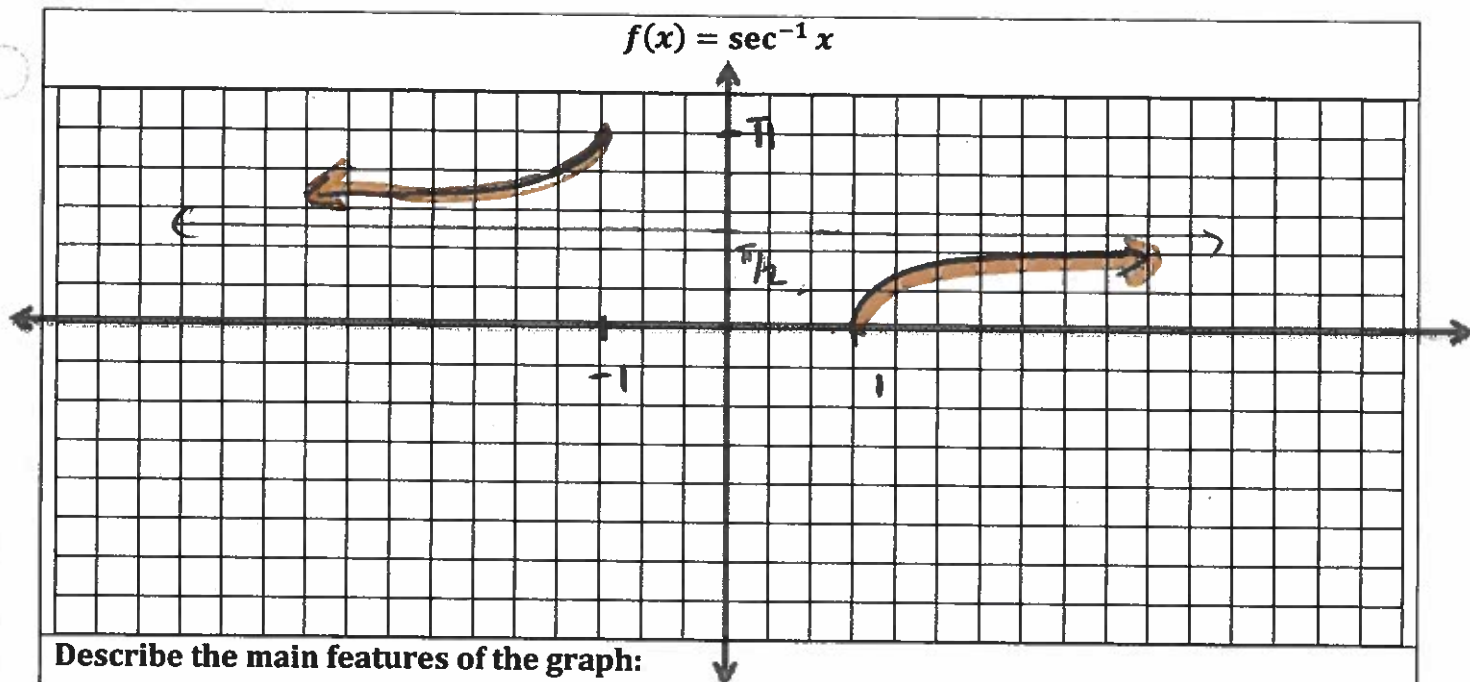
$$\sin^{-1} x = \arcsin x$$

Equation of horizontal asymptotes: none

Period: none

Domain:  $D: \{x \mid -1 \leq x \leq 1, x \in \mathbb{R}\}$

Range:  $R: \{y \mid -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, y \in \mathbb{R}\}$



Describe the main features of the graph:

General coordinates of x-intercepts:  $(1, 0)$

y-intercept: none

Maximum value:  $y = \pi$

Minimum value:  $y = 0$

$$\sec^{-1} x = \operatorname{arcsec} x$$

Equations of horizontal asymptotes: HA:  $y = \frac{\pi}{2}$

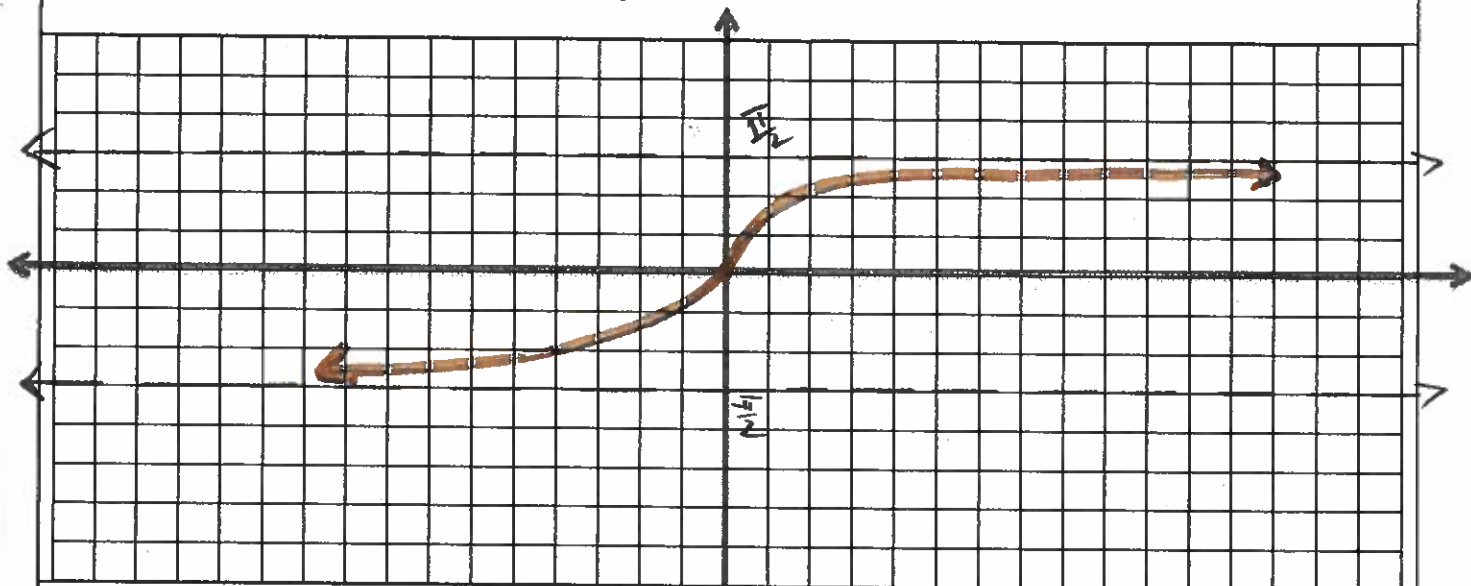
Period: none

Domain:  $D: \{x \mid x \leq -1 \text{ and } x \geq 1, x \in \mathbb{R}\}$

Range:  $R: \{y \mid 0 \leq y \leq \pi, y \neq \frac{\pi}{2}, y \in \mathbb{R}\}$



$$f(x) = \tan^{-1} x$$



Describe the main features of the graph:

General coordinates of x-intercepts:  $(0,0)$

y-intercept:  $(0,0)$

Maximum value: none

Minimum value: none

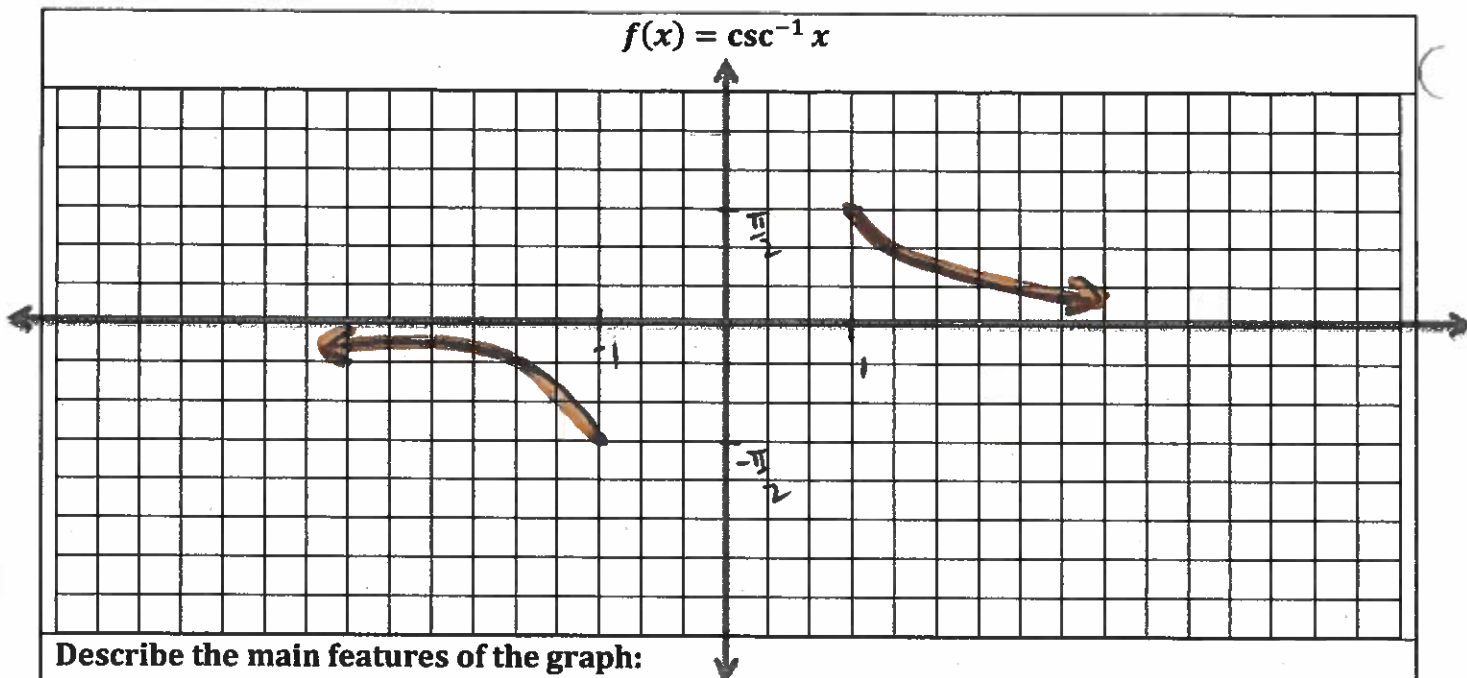
$$\tan^{-1} x = \arctan x$$

Equations of horizontal asymptotes: HA:  $y = \frac{\pi}{2}$  and  $y = -\frac{\pi}{2}$

Period: none

Domain:  $D: \{x \mid x \in \mathbb{R}\}$

Range:  $R: \{y \mid -\frac{\pi}{2} < y < \frac{\pi}{2}, y \in \mathbb{R}\}$



**Describe the main features of the graph:**

General coordinates of x-intercepts: *none*

y-intercept: *none*

Maximum value:  $y = \frac{\pi}{2}$

Minimum value:  $y = -\frac{\pi}{2}$

$$\csc^{-1} x = \operatorname{arccsc} x$$

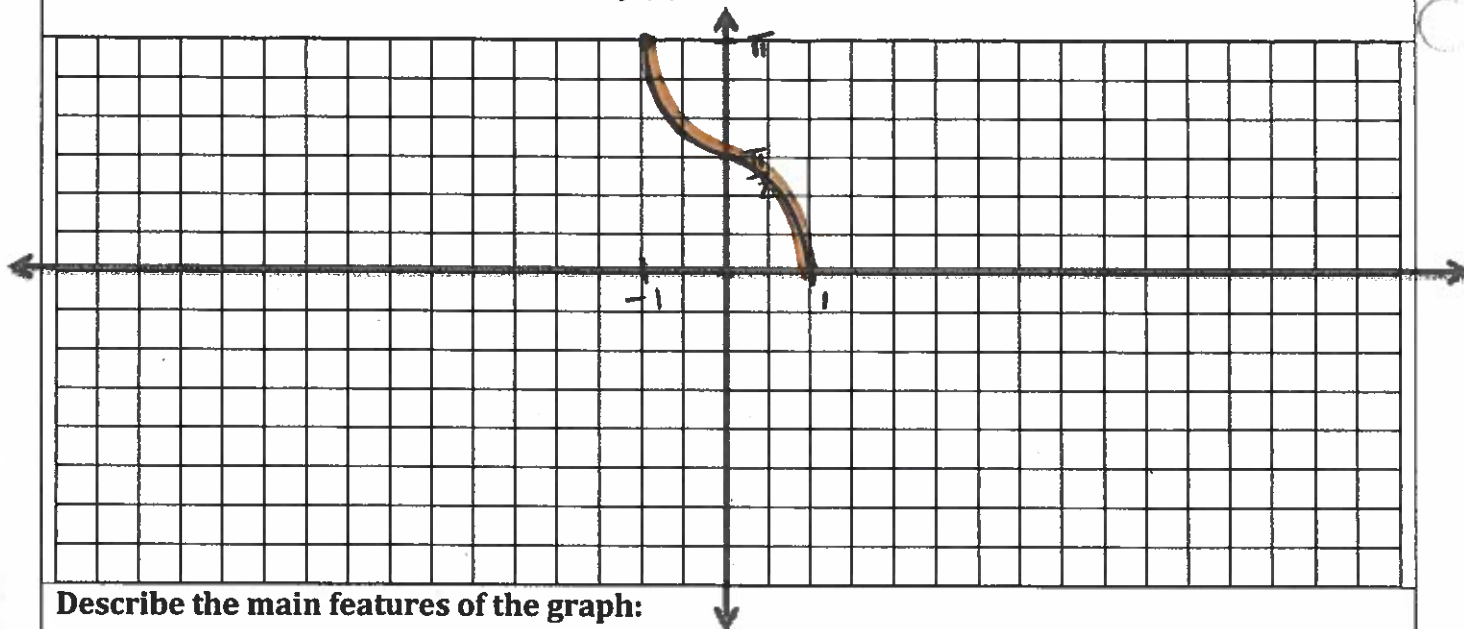
Equation~~s~~ of horizontal asymptote~~s~~: HA:  $y = 0$

Period: *none*

Domain:  $D: \{x \mid x \geq 1 \text{ and } x \leq -1, x \in \mathbb{R}\}$

Range:  $R: \{y \mid -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, y \neq 0, y \in \mathbb{R}\}$

$$f(x) = \cos^{-1} x$$



Describe the main features of the graph:

~~General coordinates of x-intercepts:~~  $(1, 0)$

y-intercept:  $(0, \frac{\pi}{2})$

Maximum value:  $y = \pi$

Minimum value:  $y = 0$

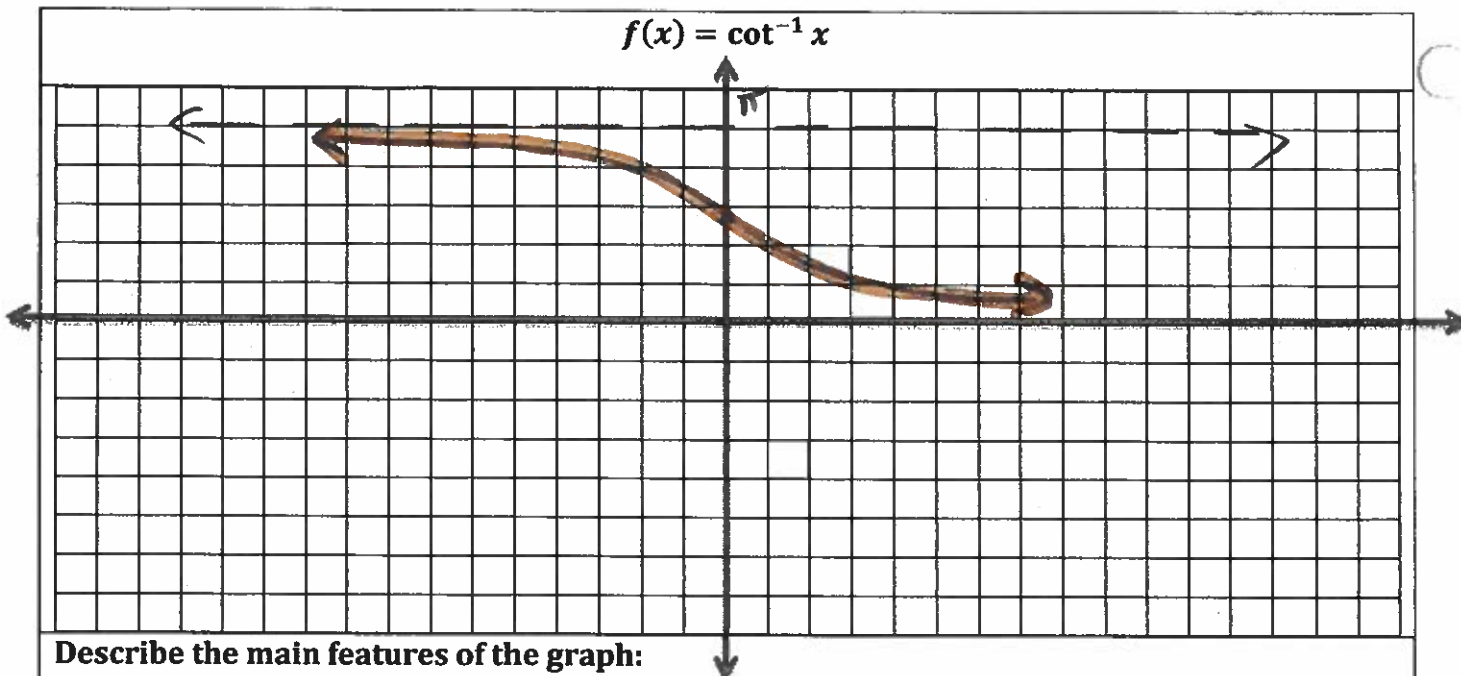
$$\cos^{-1} x = \arccos x$$

Equation of horizontal asymptotes: none

Period: none

Domain:  $D: \{x \mid -1 \leq x \leq 1, x \in \mathbb{R}\}$

Range:  $R: \{y \mid 0 \leq y \leq \pi, y \in \mathbb{R}\}$



**Describe the main features of the graph:**

General coordinates of x-intercepts: *none*

y-intercept:  $(0, \frac{\pi}{2})$

Maximum value: *none*

Minimum value: *none*

$$\cot^{-1} x = \operatorname{arccot} x$$

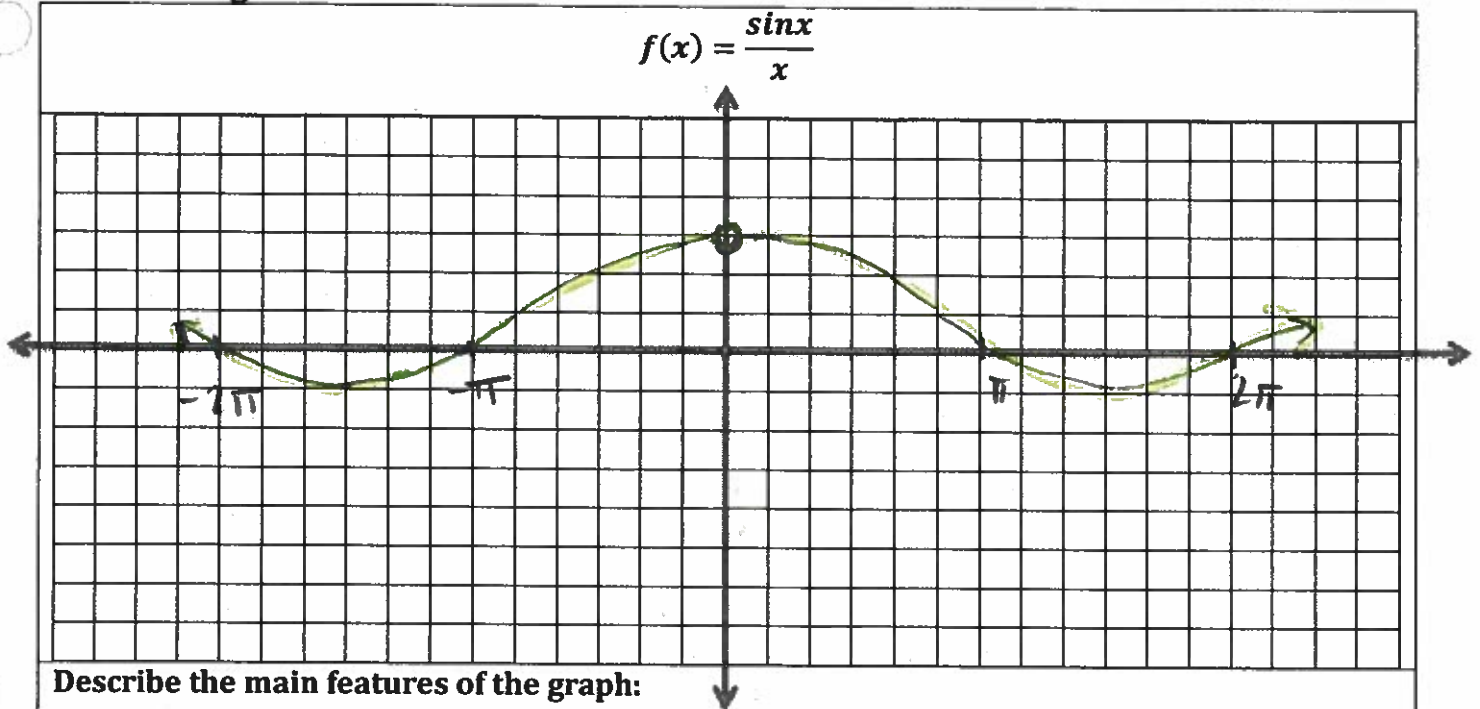
Equations of horizontal asymptotes: HA:  $y=0$  and  $y=\pi$

Period: *none*

Domain:  $D: \{x \mid x \in \mathbb{R}\}$

Range:  $R: \{y \mid 0 < y < \pi, y \in \mathbb{R}\}$

#### 4. Other Trigonometric functions:



Describe the main features of the graph:

General coordinates of x-intercepts:  $(n\pi, 0)$ , where  $n \neq 0$  and  $n \in \mathbb{Z}$

y-intercept:  $1$

Maximum value:  $y \rightarrow 1$  but not  $y = 1$

Minimum value:  $y = -0.217$

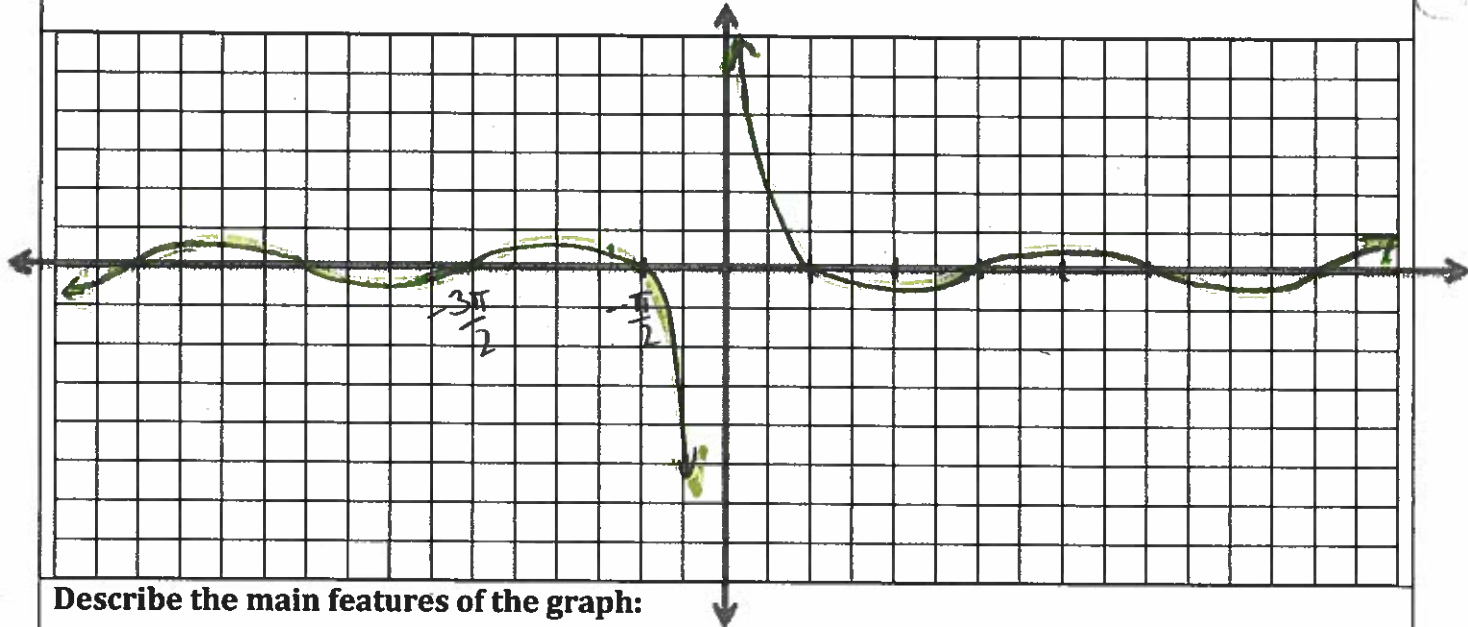
Equations of horizontal asymptotes:  $y = 0$

Period: none

Domain:  $D: \{x \mid x \neq 0, x \in \mathbb{R}\}$

Range:  $R: \{y \mid -0.217 \leq y < 1, y \in \mathbb{R}\}$

$$f(x) = \frac{\cos x}{x}$$



Describe the main features of the graph:

General coordinates of x-intercepts:  $((2n-1)\frac{\pi}{2}, 0)$

y-intercept: none

Maximum value: none

Minimum value: none

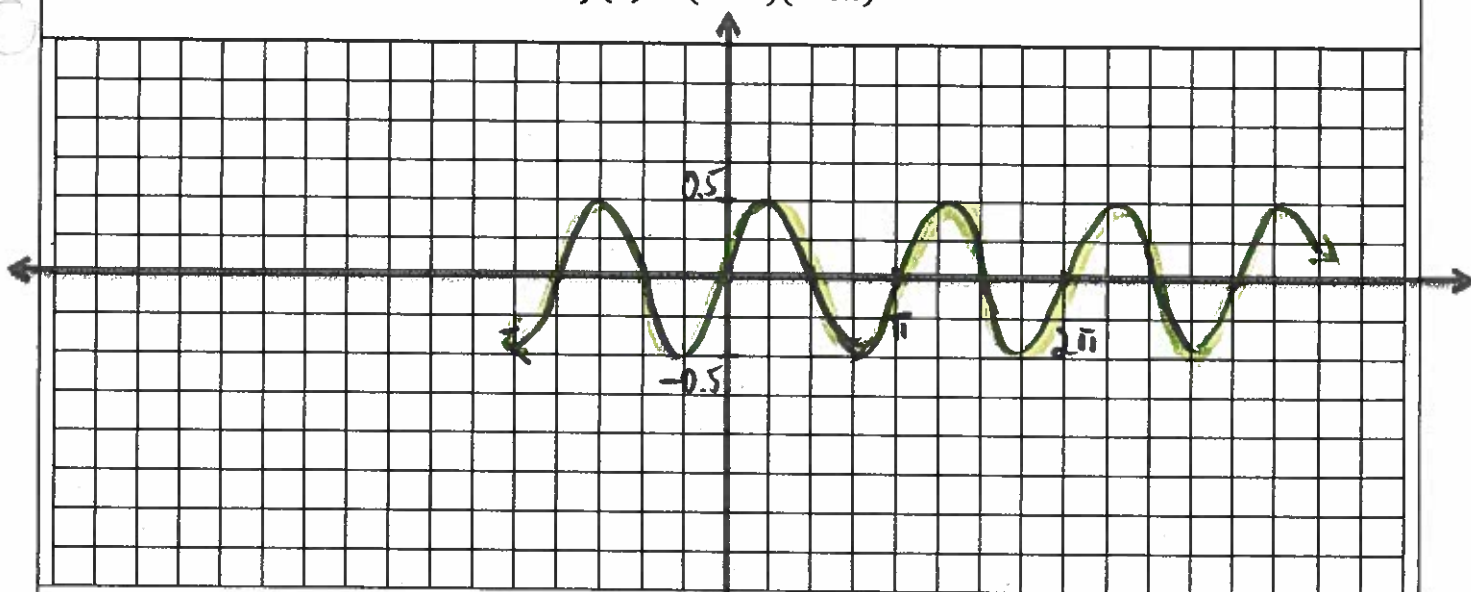
Equations of horizontal/vertical asymptotes: HA:  $y=0$  VA:  $x=0$

Period: none

Domain:  $D: \{x \mid x \neq 0, x \in \mathbb{R}\}$

Range:  $R: \{y \mid y \in \mathbb{R}\}$

$$f(x) = (\sin x)(\cos x)$$



Describe the main features of the graph:

General coordinates of x-intercepts:  $\left(\frac{n\pi}{2}, 0\right)$  where  $n \in \mathbb{Z}$

y-intercept:  $(0, 0)$

Maximum value:  $y = 0.5$

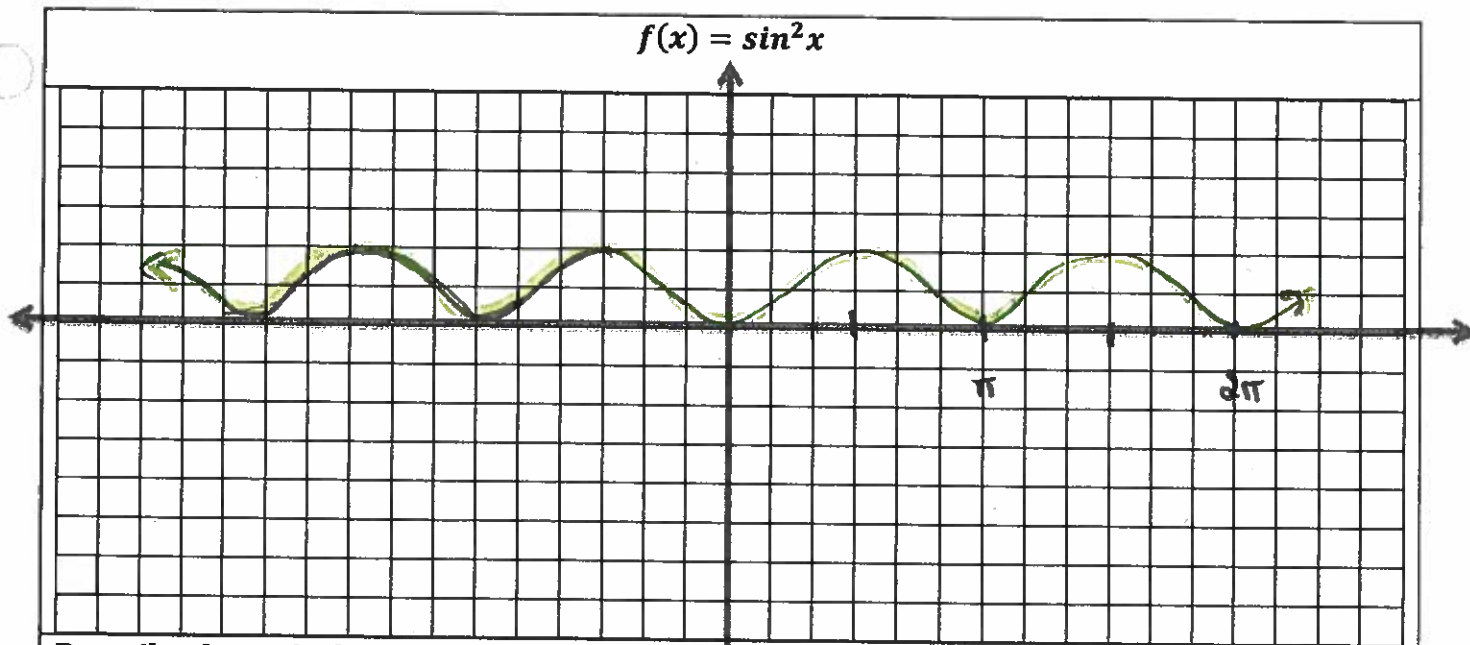
Minimum value:  $y = -0.5$

Equations of horizontal asymptotes: none

Period:  $\pi$

Domain:  $D: \{x \mid x \in \mathbb{R}\}$

Range:  $R: \{y \mid -0.5 \leq y \leq 0.5, y \in \mathbb{R}\}$



Describe the main features of the graph:

General coordinates of x-intercepts:  $(n\pi, 0)$  where  $n \in \mathbb{Z}$

y-intercept:  $(0, 0)$

Maximum value:  $y = 1$

Minimum value:  $y = 0$

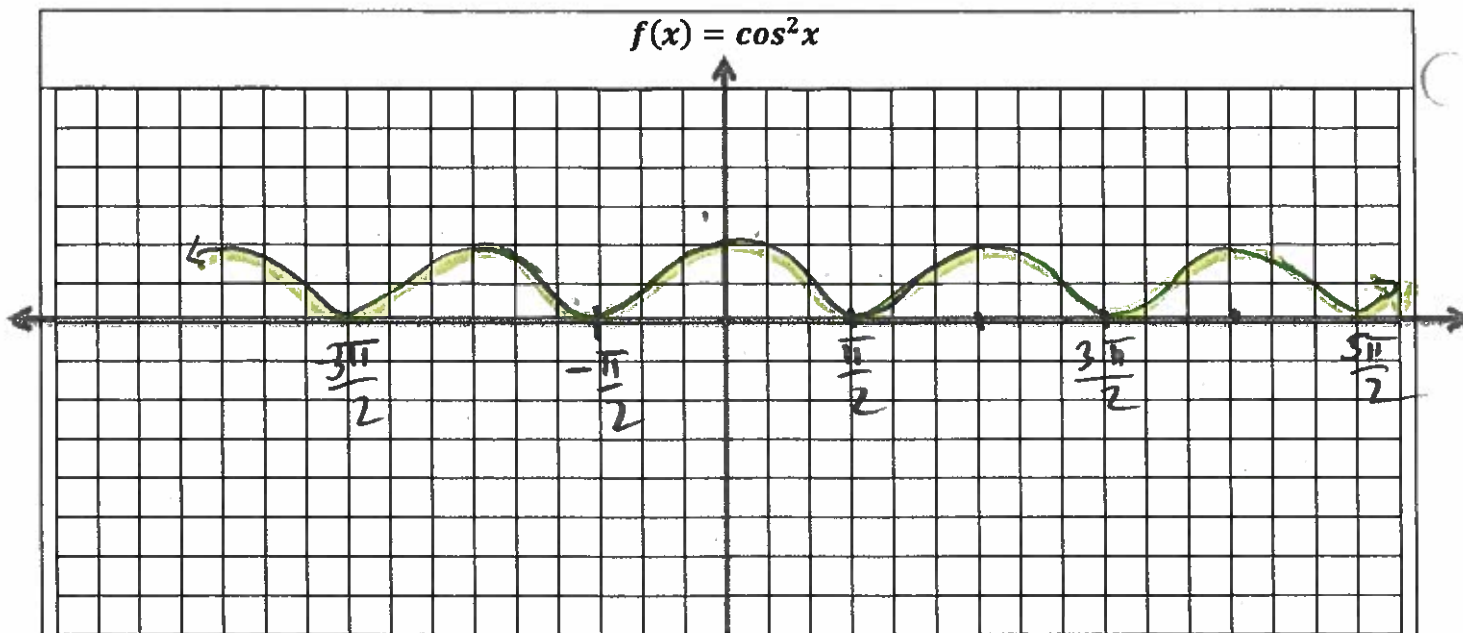
Equations of horizontal asymptotes: none

Period:  $\pi$

Domain:  $D: \{x \mid x \in \mathbb{R}\}$

Range:  $R: \{y \mid 0 \leq y \leq 1, y \in \mathbb{R}\}$





Describe the main features of the graph:

General coordinates of x-intercepts:  $\left( (2n-1)\frac{\pi}{2}, 0 \right)$

y-intercept:  $(0, 1)$

Maximum value:  $y = 1$

Minimum value:  $y = 0$

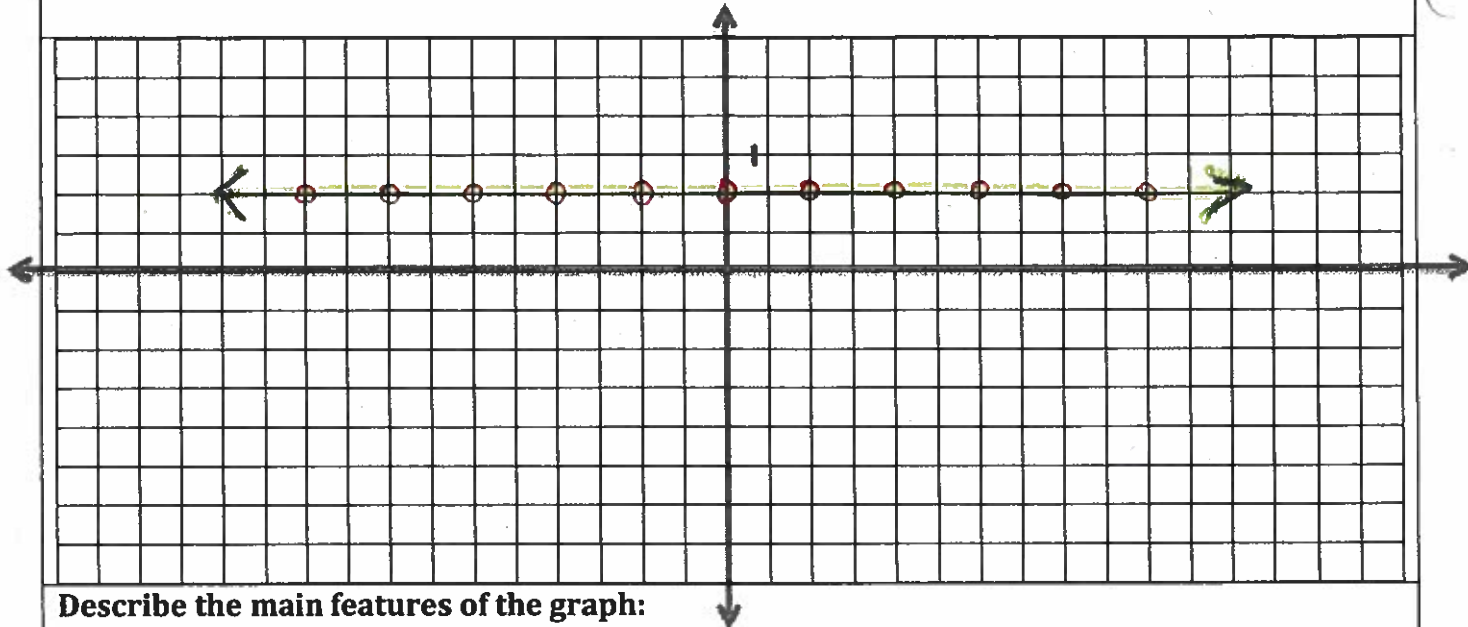
Equations of horizontal asymptotes: none

Period:  $\pi$

Domain:  $D: \{x \mid x \in \mathbb{R}\}$

Range:  $R: \{y \mid 0 \leq y \leq 1, y \in \mathbb{R}\}$

$$f(x) = (\tan x)(\cot x)$$



Describe the main features of the graph:

General coordinates of x-intercepts: none

y-intercept: none

Maximum value: none

Minimum value: none

Equations of horizontal asymptotes: none PD:  $(\frac{n\pi}{2}, 1) n \in \mathbb{Z}$

Period: none

Domain:  $D: \{x \mid x \in \mathbb{R}\}$

Range:  $R: \{y \mid y=1, y \in \mathbb{R}\}$

Arc Length:

$$s = r\theta$$

Where the angle measure is in radians.

Recall:  $\pi$  radians =  $180^\circ$

**Periodic Function:**

A function  $f(x)$  is **periodic** if there is a positive number  $p$  such that  $f(x+p)=f(x)$  for every value of  $x$ .

The smallest such value of  $p$  is the **period** of  $f$ .

| Even Trig Functions |            | Odd Trig Functions |                    |
|---------------------|------------|--------------------|--------------------|
| $\cos x$            | $\sin^2 x$ | $\sin x$           | $\csc^{-1} x$      |
| $\sec x$            | $\cos^2 x$ | $\csc x$           | $\frac{\cos x}{x}$ |
| $\cos^{-1} x$       |            | $\sin^{-1} x$      | $(\sin x)(\cos x)$ |
| $\frac{\sin x}{x}$  |            | $\tan^{-1} x$      | $\cot x$           |

$\tan x$

Transformations of Trigonometric Graphs:

$$f(x) = \pm af(\pm b(x \mp c)) + d$$