

RELATED RATES

- One of the most valuable applications of calculus is solving related rates word problems. These problems involve variables that are differentiable functions, often of time. Implicit differentiation is needed.

Solving strategy:

1. Identify all known and unknown variables.
2. Sketch and label a diagram when applicable.
3. Figure out an equation of a function that relates all known and unknown variables.
4. Use implicit differentiation to take a derivative of both sides.
5. Substitute all known values into the expression for the first derivative.
6. Determine which solution is applicable.

Example 1: Provided that radius of a circle is a function of time $r(t)$, write an equation for $A(t)$ and for $A'(t)$.

Example 2: Provided that both, radius and height of a cylinder are functions of time, write an expression for the surface area of the cylinder $S(t)$ and an expression for the volume of the cylinder $V(t)$. Differentiate both functions.

Example 3: A spherical balloon is inflated with helium at the rate $100\pi\text{ft}^3/\text{min}$.

a) How fast is the balloon's radius increasing at the instant the radius is 5 ft?

b) How fast is the surface area increasing at that instant?