C12

Reciprocal of a Function

1. Graph the original function.
2. Identify all x-intercepts of the original function (if they exist) and draw vertical asymptotes through each x-intercept.
3. Draw "helpful lines": y=1 and y=-1
4. Identify all points of intersections of the original function and the line y=1 and y=-1. These points of intersection (if they exist) are the invariant points, that is, your reciprocal will pass through those points.
5. Determine if a horizontal asymptote exists:
    - If the numerator of the reciprocal function is a real number, HA is the x-axis.

6. Sketch the reciprocal following these rules:
    - Draw through invariant points
    - Avoid asymptotes
    - Plot some helpful points (especially if invariant points do not exist) using the mapping notation for the reciprocal: $(x, y) \rightarrow \left(x, \frac{1}{y}\right)$
    - Reciprocal is decreasing where the original is increasing.
    - Reciprocal is increasing where the original is decreasing.

Practice: Sketch a graph of the following and use Desmos to check your work.

1. $f(x) = \frac{1}{x^2-4}$

2. $f(x) = \frac{1}{2x+1}$

3. $f(x) = 2csc(x)$