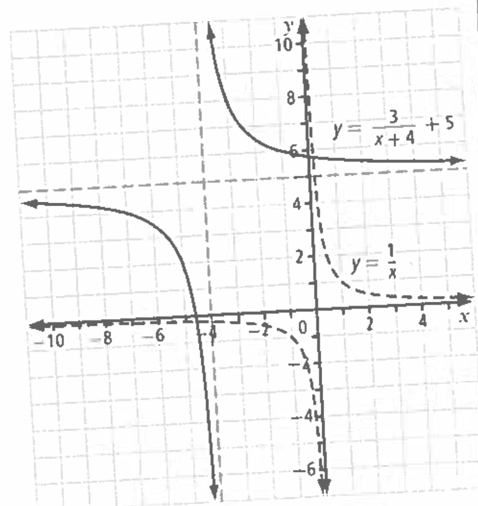
Chapter 9 Rational Functions

9.1 Exploring Rational Functions Using Transformations

KEY IDEAS

- Rational functions are functions of the form $y = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial expressions and $q(x) \neq 0$.
- You can graph a rational function by creating a table of values and then graphing the points in the table. To create a table of values,
  - identify the non-permissible value(s)
  - write the non-permissible value in the middle row of the table
  - enter positive values above the non-permissible value and negative values below the non-permissible value
  - choose small and large values of x to give you a spread of values
- You can use what you know about the base function $y = \frac{1}{x}$ and transformations to graph equations of the form $y = \frac{a}{x-h} + k$.



Example:

For $y = \frac{3}{x+4} + 5$, the values of the parameters are

$a = 3$, representing a vertical stretch by a factor of 3
$h = 4$, representing a horizontal translation 4 units to the left
$k = 5$, representing a vertical translation 5 units up
vertical asymptote: $x = -4$
horizontal asymptotes: $y = 5$

- Some equations of rational functions can be manipulated algebraically into the form $y = \frac{a}{x-h} + k$ by creating a common factor in the numerator and the denominator.

Example:

$$y = \frac{3x+6}{x-4}$$

$$y = \frac{3x-12+12+6}{x-4}$$

$$y = \frac{3x-12+18}{x-4}$$

$$y = \frac{3(x-4)}{x-4} + \frac{18}{x-4}$$

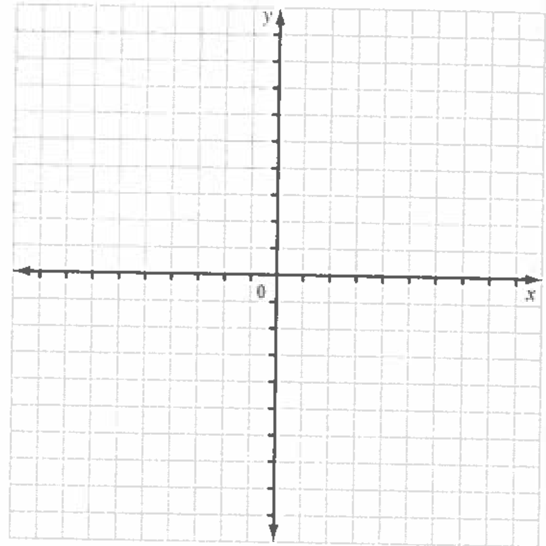
$$y = \frac{18}{x-4} + 3$$

Working Example 3: Graph a Rational Function With Linear Expressions in the Numerator and the Denominator

Graph $y = \frac{4x + 2}{x - 1}$. Identify any asymptotes and intercepts.

Solution

Let $x = 0$. Solve for y to determine the y-intercept.



The y-intercept is at (0, ________).

Let $y = 0$. Solve for x to determine the x-intercept.

$$0 = \frac{4x + 2}{x - 1}$$

$$(\_\_\_\_\_\_)(0) = (\_\_\_\_\_\_)\frac{4x + 2}{x - 1}$$

$$\_\_\_\_\_\_ = \_\_\_\_\_\_$$

$$\_\_\_\_\_\_ = 4x$$

$$\_\_\_\_\_\_ = x$$

The x-intercept is at (________, 0).

Manipulate the equation of the function algebraically to obtain the form $y = \frac{a}{x - h} + k$.

$$y = \frac{4x + 2}{x - 1}$$

$$y = \frac{4x - 4 + 4 + 2}{x - 1}$$

$y =$

> Why is 4 subtracted and added to the numerator?

> Which parameters determine the vertical and horizontal asymptotes of the transformed function?

The parameters are $a =$ ________, $h =$ ________, and $k =$ ________. State the effect of each parameter on the graph of $y = \frac{1}{x}$. Then, use the information you have generated to sketch the transformed function on the grid above.

To see a similar example, see Example 3 on pages 435–437 of *Pre-Calculus 12*.

Check Your Understanding

Practise

1. Graph each function using a table of values. Identify the asymptotes.

a) $y = \frac{6}{x}$

x	y

b) $y = \frac{-4}{x}$

x	y

2. Graph each function using transformations. Label the asymptotes and intercepts.

a) $y = \frac{2}{x + 1} - 3$

b) $y = \frac{-3}{x - 2} + 1$

3. Graph each function using technology. Identify any asymptotes and intercepts. Sketch the graph.

a) $y = \frac{2x + 1}{x - 2}$

b) $y = \frac{-3x - 4}{x + 2}$

Apply

4. Match each graph with its equation.

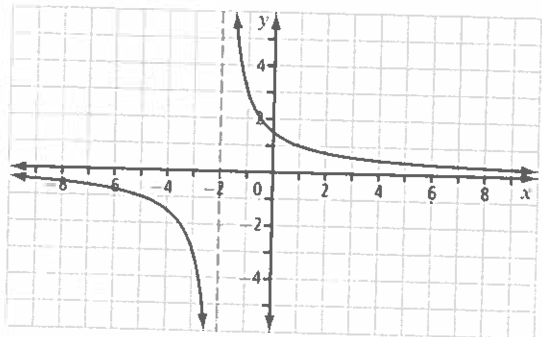
a) $y = \frac{3}{x - 2}$

b) $y = \frac{3}{x + 2}$

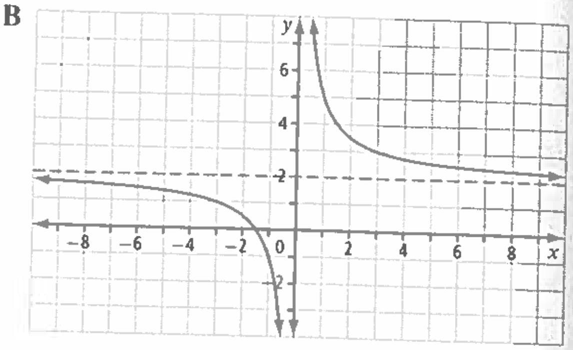
c) $y = \frac{3}{x} - 2$

d) $y = \frac{3}{x} + 2$

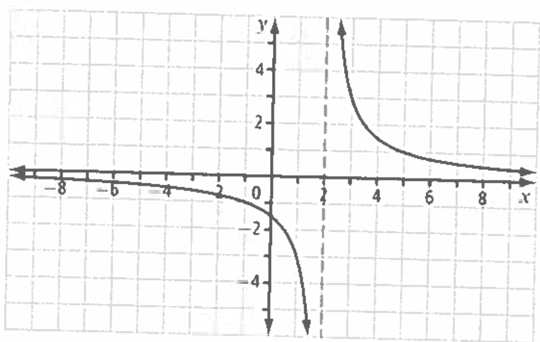
A



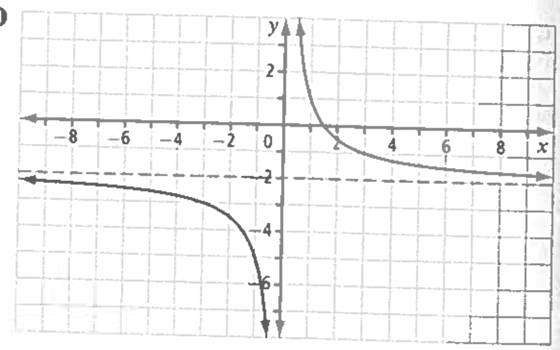
B



C

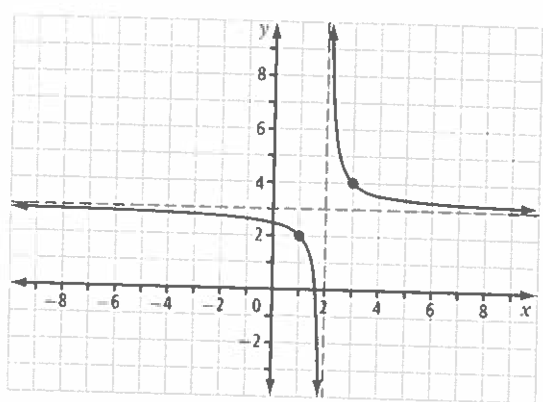


D



5. Write the equation of each function in the form of $y = \frac{a}{x - h} + k$.

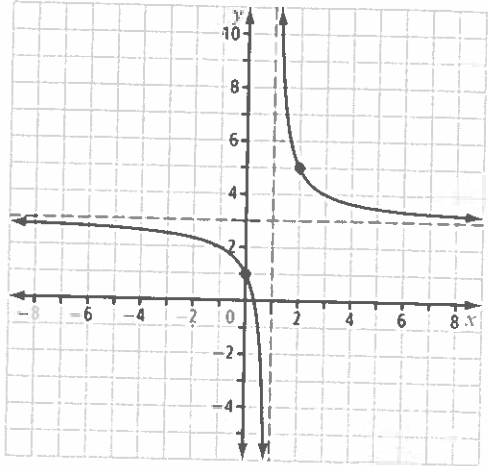
a)



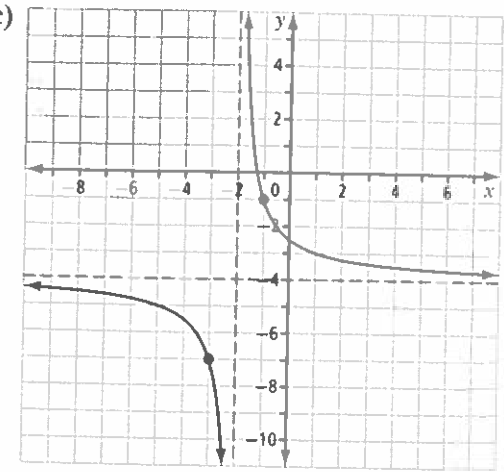
For the graph of $y = \frac{1}{x}$, what is the relationship between the intersection of the asymptotes and the point (1, 1)? How can you use this knowledge to determine a vertical stretch?

The vertical asymptote provides the ______ parameter. The horizontal asymptote provides the ______ parameter.

b)



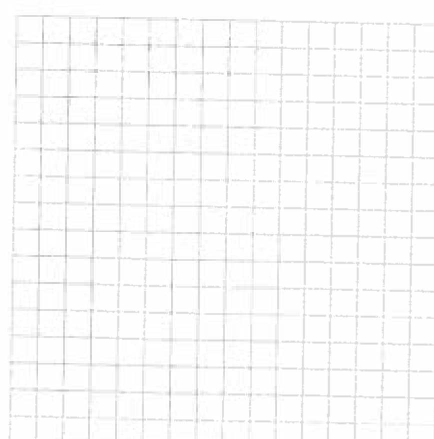
c)



6. Write each equation in the form $y = \frac{a}{x-h} + k$. Then, graph the function using transformations. Indicate the asymptotes.

a) $y = \frac{7x - 23}{x - 4}$

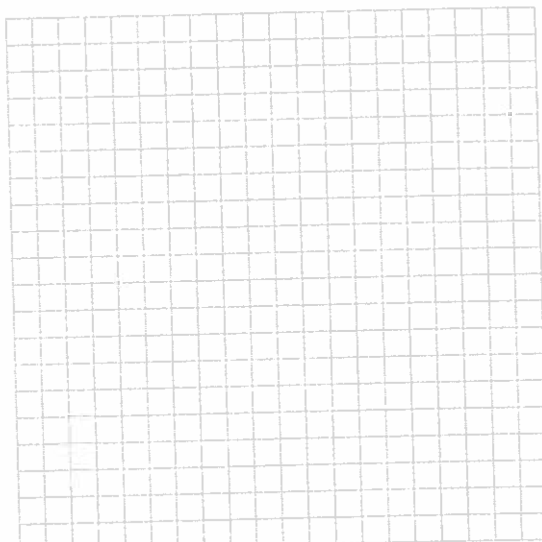
b) $y = \frac{-5x - 1}{x + 2}$



Connect

7. **a)** Determine an equation of a rational function that has an asymptote at $x = -3$ and $y = -4$. Explain the rationale for your equation.

   b) Sketch the graph of your function. Identify the asymptotes on your graph.



   c) What is the domain and range of your function?

   d) Is there another possible function with these asymptotes? Explain.

8. Describe the similarities and differences between graphing $y = \frac{2}{x-4} - 3$, $y = 2(x-4)^2 - 3$, and $y = 2\sqrt{x-4} - 3$ without technology.