C12

MVT = Mean Value Theorem for Derivatives

Theorem: If y=f(x) is continuous at every point of the closed interval [a,b] and differentiable at every point of its interior (a,b), then there is at least one point c in (a,b) at which

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Example: Consider a function $f(x) = \sqrt{x-1}$ and $1 \le x \le 3$.

a) Write an equation of a secant line AB where A (a,f(a)) and B (b,f(b)).
b) Write an equation of a tangent line to f in the interval (a,b) that is parallel to AB.

Revisiting Increasing and Decreasing Functions

Recall the definitions:

Let f be a function define on an interval I and let x_1 and x_2 be any two points in I.

1. f in increasing on I if $x_1 < x_2$ implies that $f(x_1) < f(x_2)$.
2. f is decreasing on I if $x_1 < x_2$ implies that $f(x_1) > f(x_2)$.

Note: A function that is always increasing on a an interval or always decreasing on an interval is said to be <u>monotonic</u> there.

Corollary 1: Increasing and Decreasing Functions

Let f be continuous on [a,b] and differentiable on (a,b).

1. If $f' > 0$ at each point of (a,b), then f is increasing on [a,b].
2. If $f' < 0$ at each point of (a,b), then f is increasing on [a,b].

Corollary 2: Functions with $f' = 0$ are Constant

If $f'(x) = 0$ at each point of an interval I, then there is a constant C for which $f(x) = C$ for all x in I.

Corollary 3: Functions with the Same Derivative Differ by a Constant.
If $f'(x) = g'(x)$at each point of an interval I, then there is a constant C such that $f(x) = g(x) + C$ for all x in I.

HW: p 202-203 #1-10 (light blue questions) #1-10 (dark blue questions), # 15-22,