CALCULUS 12

MEAN VALUE THEOREM

THEOREM: **MEAN VALUE THEOREM FOR DERIVATIVES**

If y=f(x) is continuous at every point of the close interval [a,b] and differentiable at every point of its interior (a,b), then there is at least one point c in the interval (a,b) at which

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

- In other words, the instantaneous rate of change at some interior point must be equal to the average rate of change over the entire interval.

Examples:

Non-examples:

Increasing and Decreasing Functions

DEFINITION: Increasing and Decreasing Function

Let f be a function defined on an interval I and let x_1 and x_2 be any two points in I.

1. f increases on I if $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$

2. f decreases on I if if $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$

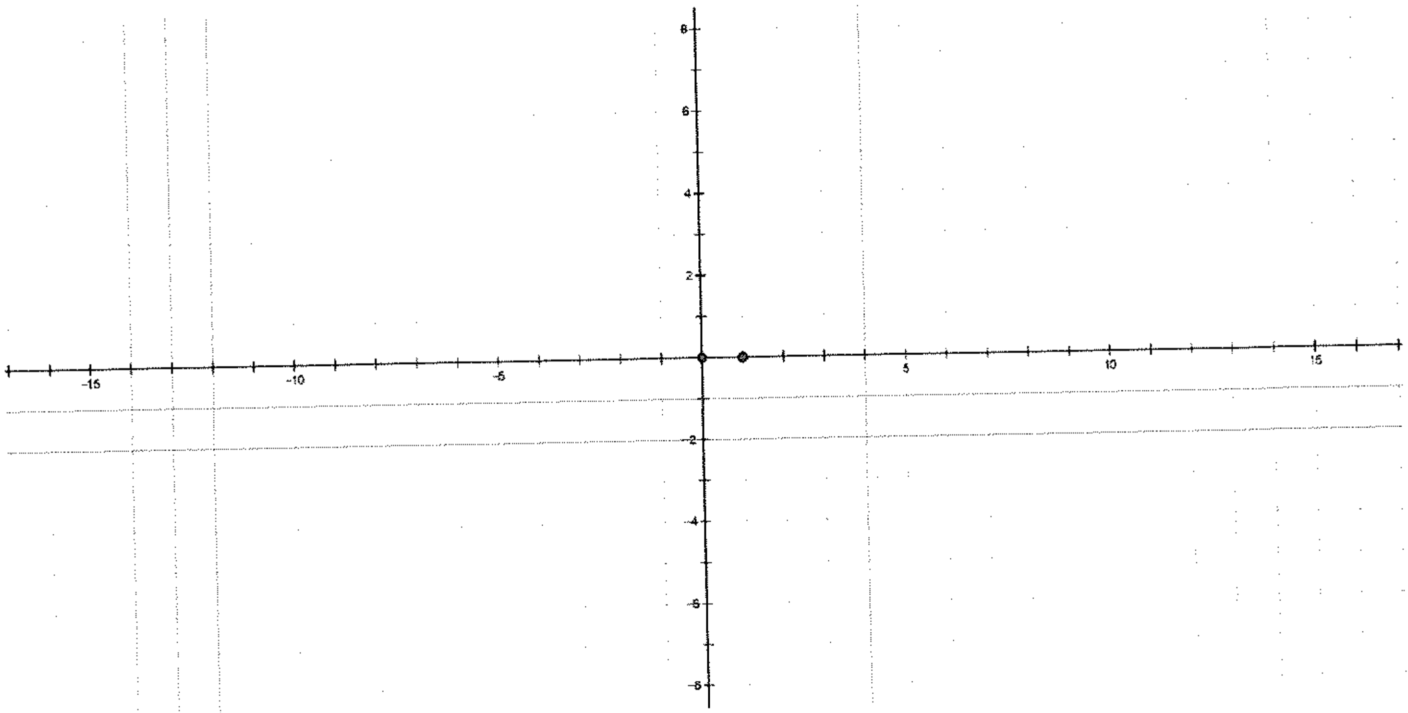
Corollary 1: Increasing and Decreasing Functions

Let f be continuous on [a,b] and differentiable on (a,b).

1. if f'> 0 at each point of (a,b), then f increases on [a,b].

2. if f'< 0 at each point of (a,b), then f decreases on [a,b].

Example: Find the intervals on which $y = x^3 - 2x + 1$ increases and on which it decreases. Sketch the graph of y and y' on a single grid.



Corollary 2: Functions with $f' = 0$ are constant

If $f'(x) = 0$ at each point of an interval I, then there is a constant C for which $f(x)= C$ or all x in the interval I.

Examples:

Corollary 3: Functions with the same derivative differ by a constant.

If $f'(x) = g'(x)$ at each point of an interval I, then there is a constant C such that $f(x) = g(x)+C$ for all x in I.

Examples:

DEFINTION: ANTIDERIVATIVE

A function F(x) is an antiderivative of a function f(x) if F'(x) = f(x) for all x in the domain of f. The process of finding an antiderivative is **antidifferentiation.**

Example: Finding a velocity function and a position function from acceleration and initial position and velocity

HW: p 202 #1-6, #9, #15-22