Kuta Software - Infinite Calculus

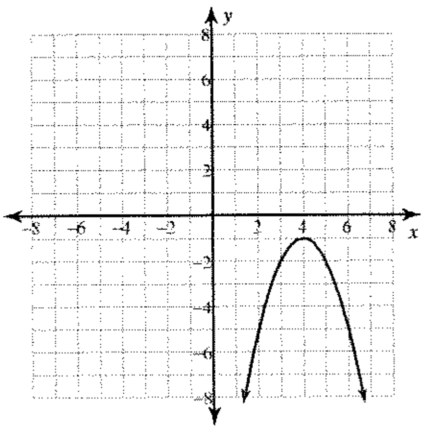
Name________________________

Mean Value Theorem

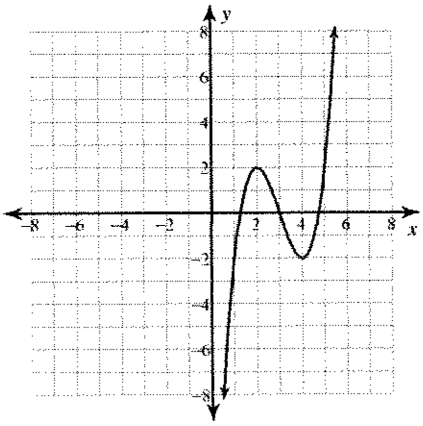
Date________________ Period____

For each problem, find the values of *c* that satisfy the Mean Value Theorem.

1) $y = -x^2 + 8x - 17$; $[3, 6]$ $\quad 9/2$



2) $y = x^3 - 9x^2 + 24x - 18$; $[2, 4]$ $\quad \frac{9 \pm \sqrt{3}}{3}$



3) $y = -\frac{x^2}{2} + x - \frac{1}{2}$; $[-2, 1]$ $\quad -1/2$

4) $y = \frac{x^2}{2} - 2x - 1$; $[-1, 1]$ $\quad 0$

5) $y = x^3 + 3x^2 - 2$; $[-2, 0]$ $\quad \frac{-3 \pm \sqrt{3}}{3}$

6) $y = -x^3 + 4x^2 - 3$; $[0, 4]$ $\quad 8/3$

7) $y = \frac{x^2 - 9}{3x}$; $[1, 4]$ $\quad 2$

8) $y = \frac{x^2}{2x - 4}$; $[-4, 1]$ $\quad 2 - \sqrt{6}$

9) $y = -(-2x+6)^{\frac{1}{2}}$; [−2, 3]   7/4

10) $y = -(-5x+25)^{\frac{1}{2}}$; [3, 5]   9/2

For each problem, determine if the Mean Value Theorem can be applied. If it can, find all values of c that satisfy the theorem. If it cannot, explain why not.

11) $y = -\dfrac{x^2}{4x+8}$; [−3, −1]   dis cont.

12) $y = \dfrac{-x^2+9}{4x}$; [1, 3] $\sqrt{3}$

13) $y = -(6x+24)^{\frac{2}{3}}$; [−4, −1] $-\dfrac{28}{9}$

14) $y = (x-3)^{\frac{2}{3}}$; [1, 4]   not different.

Critical thinking question: Bonus for marks ☺

15) Use the Mean Value Theorem to prove that $|\sin a - \sin b| \le |a-b|$ for all real values of a and b where $a \ne b$.

Kuta Software - Infinite Calculus

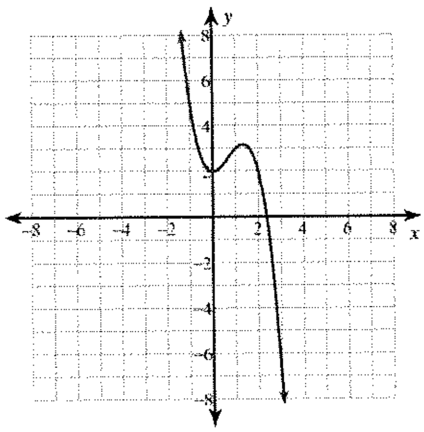
Name____________________________________

Intervals of Increase and Decrease

Date________________ Period____

For each problem, find the x-coordinates of all critical points, find all discontinuities, and find the open intervals where the function is increasing and decreasing.

1) $y = -x^3 + 2x^2 + 2$



$x = 0,\ x = \frac{4}{3}$

$\uparrow (0, \frac{4}{3}),\ \downarrow (-\infty, 0)$

$\downarrow (\frac{4}{3}, \infty$

2) $y = x^3 - 11x^2 + 39x - 47$

$x = 3,\ x = \frac{13}{3}$

$\uparrow (-\infty, 3),\ (\frac{13}{3}, \infty) \downarrow ($

3) $y = -x^4 + 3x^2 - 3$

$x = \frac{\pm\sqrt{6}}{2},\ x = 0$

$\uparrow (-\infty, -\frac{\sqrt{6}}{2}),\ \uparrow (0, \frac{\sqrt{6}}{2}$

$\downarrow (-\frac{\sqrt{6}}{2}, 0),\ \downarrow (\frac{\sqrt{6}}{2}, \infty$

4) $y = \frac{x^2}{4x + 4}$

$x = -2, x = 0$ disc. $x = -1$

$\uparrow (-\infty, 2),\ (0, \infty)$

$\downarrow (-2, -1),\ (-1, 0)$

5) $y = \frac{3x^2 - 3}{x^3}$

$x = \pm\sqrt{3}$; disc. $x = 0$

$\uparrow$ $(-\sqrt{3}, 0), (0, \sqrt{3})$

$\downarrow$ $(-\infty, -\sqrt{3}), (\sqrt{3}, \infty)$

6) $y = (2x - 8)^{\frac{2}{3}}$

$x = 4$

$\uparrow$ $(4, \infty)$, $\downarrow$ $(-\infty, 4)$

7) $y = -\frac{1}{5}(x-4)^{\frac{5}{3}} - 2(x-4)^{\frac{2}{3}} - 1$

$x = 0$, $x = 4$

$\uparrow$ $(0, 4)$, $\downarrow$ $(-\infty, 0), (4, \infty)$

Critical thinking question: Bonus for extra marks.

8) If functions f and g are increasing on an interval, show that $f + g$ is increasing on the same interval.

9) Give an example where functions f and g are increasing on the interval $(-\infty, \infty)$, but where $f - g$ is decreasing.

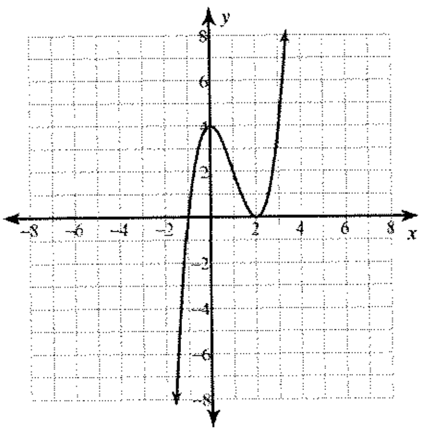
Kuta Software - Infinite Calculus Name________________________________

Intervals of Concavity

Date________________ Period____

For each problem, find the x-coordinates of all points of inflection, find all discontinuities, and find the open intervals where the function is concave up and concave down.

1) $y = x^3 - 3x^2 + 4$



$x = 1$

$\cup\ (1, \infty),\ \cap\ (-\infty, 1)$

2) $y = x^3 - 2x^2 - 2$

$x = \frac{2}{3}$

$\cup\ (\frac{2}{3}, \infty),\ \cap\ (-\infty, \frac{2}{3})$

3) $y = x^4 + x^3 - 3x^2 + 1$

$x = -1,\ x = \frac{1}{2}$

$\cup\ (-\infty, -1),\ (\frac{1}{2}, \infty)$

$\cap\ (-1, \frac{1}{2})$

4) $y = \frac{1}{x - 3}$

no inflectionpoints, disc. $x = 3$

$\cup\ (3, \infty),\ \cap\ (-\infty, 3)$

5) $y = -\dfrac{x^3}{x^2 - 4}$

$x = 0$, disc. $x = \pm 2$
$\cup$ $(-\infty, -2)$, $(0, 2)$
$\cap$ $(-2, 0]$, $(2, \infty)$

6) $y = (5x + 30)^{\frac{2}{3}}$

no inflection points
$\cup$ never
$\cap$ $(-\infty, -6)$, $(-6, \infty)$

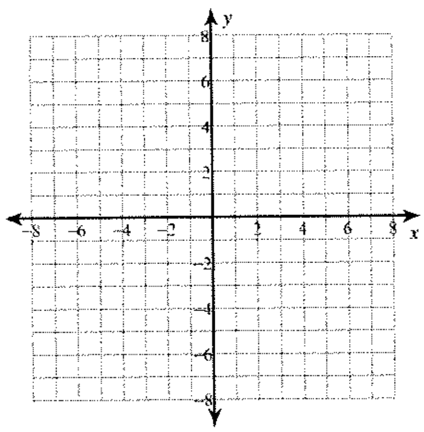
7) $y = -\dfrac{3}{16}(x-1)^{\frac{4}{3}} - \dfrac{3}{2}(x-1)^{\frac{1}{3}} + 2$

$x = 1$, $x = 5$
$\cup$ $(1, 5)$, $\cap$ $(-\infty, 1)$, $(5, \infty)$

Critical thinking question:

Bonus for extra marks ☺

8) Sketch a continous curve $y = f(x)$ where $f(1) = 0$, $f'(0) = 0$, $f'(2) = 0$, $f''(x) < 0$ for $x < 1$, and $f''(x) > 0$ for $x > 1$.



Kuta Software - Infinite Calculus

Name________________________________

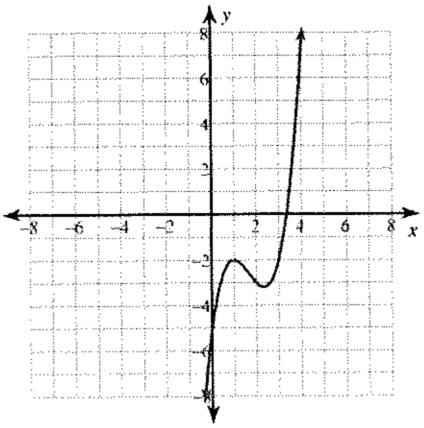
Assignment

Date________________ Period____

= local

For each problem, find all points of relative minima and maxima.

1) $y = x^3 - 5x^2 + 7x - 5$

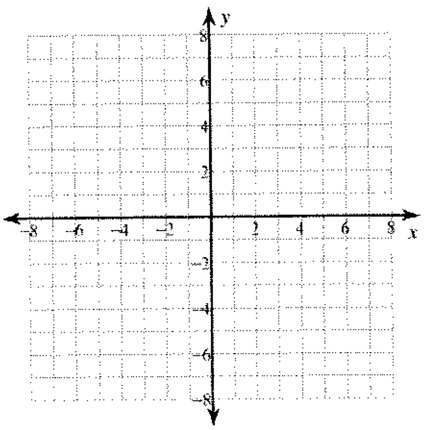


L. min $\left(\frac{7}{3}, -\frac{86}{27}\right)$

L. max $(1, -2)$

For each problem, find all points of relative minima and maxima. You may use the provided graph to sketch the function.

2) $y = x^3 - 6x^2 + 9x + 1$



L. min $(3, 1)$

L. max $(1, 5)$

For each problem, find all points of relative minima and maxima.

3) $y = -x^3 - 3x^2 - 1$

$(-2, -5)$
$(0, -1)$

4) $y = x^4 - 2x^2 + 3$

$(-1, 2), (1,$
$(0, 3)$

5) $y = x^4 - x^2$

$\left(-\frac{\sqrt{2}}{2}, -\frac{1}{4}\right), \left(\frac{\sqrt{2}}{2}, -\frac{1}{4}\right)$
$(0, 0)$

6) $y = -\frac{2}{x^2 - 4}$

$(0, \frac{1}{2})$
no max.

7) $y = (2x - 8)^{\frac{2}{3}}$

$(4, 0)$
no l. max

8) $y = -\frac{1}{5}(x - 4)^{\frac{5}{3}} - 2(x - 4)^{\frac{2}{3}}$

$\left(0, \frac{-12\sqrt[3]{2}}{5}\right)$
$(4, 0)$

Critical thinking questions: Bonus for extra marks.

9) Give an example function $f(x)$ where $f''(0) = 0$ and there is no relative minimum or maximum at $x = 0$.

10) Give an example function $f(x)$ where $f''(0) = 0$ and there is a relative maximum at $x = 0$.