#3 $y = \log_2 \sqrt[3]{x}$

$(2^y)^3 = (\sqrt[3]{x})^3$

$2^{3y} = x$

Logarithms - Test

Name: Key **Date:**____________

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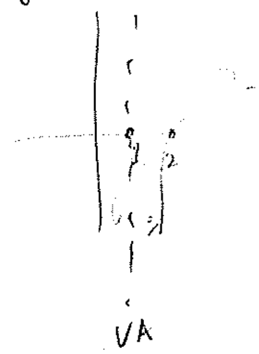
Multiple Choice

[12] ***For #1 to 6, select the best answer.***

1. The graph of $f(x) = \log_b x$, $b > 1$, is translated such that the equation of the new graph is expressed as $y - 2 = f(x - 1)$. The domain of the new function is

$y = f(x-1) + 2$ $\quad (x, y) \to (x+1, y+2)$



A $\{x \mid x > 0, x \in R\}$ **C** $\{x \mid x > 2, x \in R\}$

B $\{x \mid x > 1, x \in R\}$ **D** $\{x \mid x > 3, x \in R\}$

2. The x-intercept of the function $f(x) = \log_5 x + 3$ is

A 5^{-3} **B** 0
C 1 **D** 5^3

$y = \log_5 x + 3$

$0 = \log_5 x + 3\log_5 5$

$0 = \log_5 (x \cdot 5^3)$

$0 = \log_5 125x$

$5^0 = 125x \to 1 = 125x \to x = \frac{1}{125}$

$x = \frac{1}{5^3} = 5^{-3}$

3. The equation $y = \frac{1}{3}\log_2 x$ can also be written as

A $y = 2^{\frac{x}{3}}$ **B** $x = 2^{\frac{y}{3}}$
C $2^{3x} = y$ **D** $2^{3y} = x$

4. The range of the inverse function, f^{-1}, of $f(x) = \log_4 x$, is

$x = \log_4 y \quad 4^x = y$

(A) $\{y \mid y > 0, y \in R\}$ **C** $\{y \mid y \geq 0, y \in R\}$

B $\{y \mid y < 0, y \in R\}$ **D** $\{y \mid y \in R\}$

5. A graph of the function $y = \log_3 x$ is transformed. The image of the point (3, 1) is (6, 3). The equation of the transformed function is

$(x, y) \to (2x, y+2)$ OR $\to (x+3, 3y)$

A $y = 3\log_3 (x - 3)$ **C** $y - 3 = \log_3 (x - 3)$

B $y = 3\log_3 (x + 3)$ **D** $y + 3 = \log_3 (x + 3)$

6. If $\log_{27} x = y$, then $\log_9 x$ equals

$27^y = x \quad \log_9(27^y) = \log_9 3^{3y}$

$= \log_9 9^y + \log_9 3^y$

$= y + \frac{y}{2}$

$= \frac{2y+y}{2} = \frac{3y}{2}$

A $\frac{3y}{2}$ **B** $\frac{2y}{3}$
C $3y$ **D** 4^y

Short Answer

[3] **7.** If $\log_3 5 = x$, express $2\log_3 45 - \frac{1}{2}\log_3 225$ in terms of x.

$3^x = 5$

$2(\log_3 3^2 + \log_3 5) - \log_3 \sqrt{225}$

$2(2 + \log_3 3^x) - \log_3 15$

$= 2(2 + x) - [\log_3 3 + \log_3 5]$

$= 4 + 2x - [1 + \log_3 3^x]$

$= 4 + 2x - 1 - x$

$= \boxed{x + 3}$

[6] **8.** Determine the value of x algebraically.

a) $\log_4 x = -3$

$4^{-3} = x$

$x = \frac{1}{4^3}$

$x = \frac{1}{64}$

b) $\log_x 64 = \frac{2}{3}$

$(\sqrt[3]{x^2})^3 = (64)^3$

$x^2 = 262144$

$x = 512$

c) $5^{\log_5 25} = x$

$\log_5 x = \log_5 25$

$\boxed{x = 25}$

[2] **d)** $\log_3 (x+1)^2 = 2$

$3^2 = (x+1)^2$

$9 = x^2 + 2x + 1$

$0 = x^2 + 2x - 8$

$(x-2)(x+4) = 0$

$x = 2, -4$

e) $\log_2 (\log_x 256) = 3$

$2^3 = \log_x 256$

$8 = \log_x 256$

$x^8 = 256$

$x = \sqrt[8]{256}$

$\boxed{x = 2}$

[9] **9.** Solve for x. **CHECK YOUR ANSWERS.** Clearly identify all valid answers.

a) $\log(2x-3) + \log(x-2) = \log(2x-1)$

$\log(2x-3)(x-2) = \log(2x-1)$

$\log(2x^2-7x+6) - \log(2x-1) = 0$

$\log\left(\frac{2x^2-7x+6}{2x-1}\right) = 0$

$10^0 = \frac{2x^2-7x+6}{2x-1}$

$2x-1 = 2x^2-7x+6$

$0 = 2x^2-9x+7$

$0 = (x-1)(2x-7)$

$x = 1, \frac{7}{2}$

$\log[2(1)-3] = \log -1$ → not defined

$x = 1$ not a solution

$\boxed{\therefore x = 3.5}$

b) $\log(x-7) - \log(x-3) = \log(2x+1)$

$\log\frac{(x-7)}{(x-3)} = \log(2x+1)$

$\frac{x-7}{x-3} = 2x+1$

$x-7 = (2x+1)(x-3)$

$x-7 = 2x^2-6x+x-3$

$0 = 2x^2-6x+4$

$0 = 2x^2-2x-4x+4$

$0 = (2x-4)(x-1)$

$2x-4=0$

$2x=4$

$x=2$

$x-1=0$

$x=1$

Check: $\log(2-7)$ → not defined

$\log(1-7)$ → not defined

⇒ no [illegible]

Check:
$\log(2-4) \rightarrow ND$
$\log_2(8-4) \rightarrow \checkmark$
$\therefore x = 8$

c) $2\log_2(x-4) - \log_2 x = 1$

$\log_2(x-4)^2 - \log_2 x = 1$

$\log_2\left(\frac{(x-4)^2}{x}\right) = 1$

$2^1 = \frac{(x-4)^2}{x}$

$2x = x^2 - 8x + 16$

$0 = x^2 - 10x + 16$

$0 = (x-2)(x-8)$

$x = 2 \; ; \; x = 8$

[2] **10.** The point $(6, -4)$ lies on the graph of $y = \log_b x$. Determine the value of b to the nearest tenth.

$-4 = \log_b 6$

$b^{-4} = 6$

$\frac{1}{b^4} = 6$

$6b^4 = 1$

$\sqrt[4]{b} = \sqrt[4]{\frac{1}{6}}$

$b = 0.6$

Extended Response

[2] **11.** Solve the equation $5^x = 104$, **graphically and algebraically**. Round your answer to the nearest hundredth.

OK $\log 5^x = \log 104$

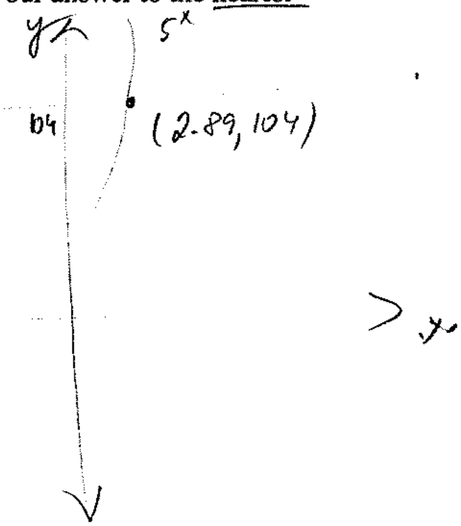
$x \cdot \log 5 = \log 104$

$x = \frac{\log 104}{\log 5}$

$\log_5 104 = x$

$x = \frac{\log 104}{\log 5}$

$x \doteq 2.89$



[4] **12.** Given $f(x) = \log_3 x$ and $g(x) = \log_3 9x$.

a) Describe the transformation of $f(x)$ required to obtain $g(x)$ as a stretch. Horizontal stretch by a factor $\frac{1}{9}$.

b) Describe the transformation of $f(x)$ required to obtain $g(x)$ as a translation. $\log_3 9 + \log_3 x = 2 + \log_3$

$\Rightarrow$ Vertical translation up by 2.

c) Determine the x-intercept of $f(x)$. How can the x-intercept of $g(x)$ be determined using your answer to parts a) or b)?

$0 = \log_3 x$

$3^0 = x$

$f(x)$: $\boxed{x = 1}$

x-int: $(1, 0)$

a) $(x, y) \rightarrow \left(\frac{x}{9}, y\right)$ so: $(1, 0) \rightarrow \left(\frac{1}{9}, 0\right)$

b) $(x, y) \rightarrow (x, y+2)$

$y + 2 = 0$

$y = -2$

$-2 = \log_3 x \Rightarrow x = 3^{-2} = \frac{1}{9}$

[4] **13.** Explain how the graph of $y = \frac{\log_4 (3x-1)}{2} + 1$ can be generated by transforming the graph of $y = \log_4 x$.

$y = \frac{1}{2} \log_4 (3x-1) + 1$

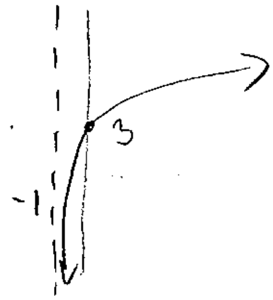
$y = \frac{1}{2} \log_4 [3(x - \frac{1}{3})] + 1$

- VS by a factor of $\frac{1}{2}$
- HS by a factor of $\frac{1}{3}$
- HT right by $\frac{1}{3}$
- VT up by 1

[5] **14.** Identify the following characteristics of the graph of the function $y = 2 \log_4 (x+1) + 3$.

a) the equation of the asymptote

$x = -1$



b) the domain and range

$D: \{x \mid x > -1, x \in \mathbb{R}\}$

$R: \{y \mid y \in \mathbb{R}\}$

$y = 2 \log_4 (0+1) + 3$
$y = 2 \log_4 1 + 3$
$y = \log_4 1 + 3$
$y = 0 + 3$

$\boxed{\therefore y = 3}$

$(0, 3)$

c) the x-intercept and the y-intercept

$0 = 2 \log_4 (x+1) + 3$
$-3 = 2 \log_4 (x+1)$
$-\frac{3}{2} = \log_4 (x+1)$

$4^{-\frac{3}{2}} = x+1$

$\frac{1}{(\sqrt{4})^3} = x+1$

$\frac{1}{8} = x+1$

$\frac{1}{8} - \frac{8}{8} = x$

$\boxed{x = \frac{-7}{8}}$

$(-\frac{7}{8}, 0)$

[3] **15.** An investment of \$2000 pays interest at a rate of 3.5% per year. Determine the number of months required for the investment to grow to at least \$3000 if interest is compounded monthly.

$A = 2000 \left(1 + \frac{0.035}{12}\right)^n$

$3000 = 2000 (1.00292)^n$

$\frac{3}{2} = 1.00292^n$

$\log 1.5 = \log 1.00292^n$

$n = \frac{\log 1.5}{\log 1.00292}$

$n = 139.06$ months

$\therefore$ It will take 140 months

16. Radioactive iodine-131 has a half-life of 8.1 days. How long does it take for the level of radiation to reduce to 1% of the original level? Express your answer to the <u>nearest tenth</u>.

[3]

$$0.01 = \left(\frac{1}{2}\right)^{\frac{n}{8.1}}$$

$$(0.01)^{8.1} = \left(\sqrt[8.1]{\frac{1}{2}^{n}}\right)^{8.1}$$

$$6.31 \times 10^{-17} = \frac{1}{2}^{n}$$

$$\log_{\frac{1}{2}} 6.31 \times 10^{-17} = n$$

$$n = 53.8$$

∴ It would take 53.8 days.

OR

$$A = 1\left(\frac{1}{2}\right)^{\frac{t}{8.1}}$$

$$0.01 = 1\left(\frac{1}{2}\right)^{\frac{t}{8.1}}$$

$$\log 0.01 = \frac{t}{8.1} \cdot \log \frac{1}{2}$$

$$t = \left(\frac{\log 0.01}{\log \frac{1}{2}}\right)(8.1)$$

$$t = 53.8 \text{ days}$$

∴ It would take 53.8 days.