CALCULUS 12

LINEARIZATION

Recall: every differentiable function is locally linear.

DEFINTION: **Linearization**

If f is differentiable at x=a, then the equation of the tangent line,

L(x) = f(a) + f'(a)(x-a), defines the linearization of f at a.

The approximation f (x)≈L(x) is the standard linear approximation of f at a. The point x = a is the center of the approximation.

Example1: Find the standard linearization of $\sqrt{1+x}$ at x=0 and use the expression to estimate $\sqrt{1.02}$

$L(x) = f(a) + f'(a)(x-a)$

$= \sqrt{1+0} + \frac{1}{2\sqrt{1+0}}(x-0)$

$= 1 + \frac{1}{2}x$

$\sqrt{1.02} \doteq L(1.02)$ $a = 0.02$

$L(0.02) = 1 + \frac{1}{2}(0.02)$

$= 1 + \frac{0.02}{2}$

$= 1 + 0.01$

$= 1.01$

Example2: Find the standard linearization of f(x)=cosx at x=$\frac{\pi}{2}$ and use the expression approximate cos 1.75.

$L(x) = f(a) + f'(a)(x-a)$; $a = \frac{\pi}{2}$

$= \cos\frac{\pi}{2} + (-\sin\frac{\pi}{2})(x - \frac{\pi}{2})$

$= 0 - 1(x - \frac{\pi}{2})$

$= -x + \frac{\pi}{2}$

$\cos(1.75) \doteq L(1.75)$

$\doteq (-1.75) + \frac{\pi}{2}$

$= -0.179$

Example 3: **Approximating Binomial Powers**

$$(1+x)^k \approx 1+kx$$

Where k is a real number and x is close to 0

Approximate $\frac{1}{\sqrt{1-x^2}}$ $= (1-x^2)^{-\frac{1}{2}}$

$\doteq 1 + (-\frac{1}{2})(-x^2)$

$\doteq 1 + \frac{x^2}{2}$

$f'(x) = \frac{1}{3}(x)^{-\frac{2}{3}}$

Example 4: Linear approximation of radical expressions. Use standard linearization to approximate

a) $\sqrt{123} \doteq \sqrt{121}$ let $a = 121$

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$L(a) = f(a) + f'(a)(x-a)$

$= \sqrt{121} + \frac{1}{2\sqrt{121}}(123-121)$

$= 11 + \frac{1}{22}(2)$

$= 11 + \frac{2}{22}$

$= 11 + \frac{1}{11}$

$= 11.\overline{09}$

b) $\sqrt[3]{123} \doteq \sqrt[3]{125}$ let $a = 125$

$L(a) = f(a) + f'(a)(x-a)$

$= \sqrt[3]{125} + \frac{1}{3}(125)^{-\frac{2}{3}}(123-125)$

$= 5 + \frac{1}{3}(5)^{-2}(-2)$

$= 5 + \frac{1}{75}(-2)$

$= 5 + (-\frac{2}{75})$

$= 4.97\overline{3}$

HW: p242-243 #1-14