

LINEARIZATION

Recall: every differentiable function is locally linear.

DEFINITION: Linearization

If f is differentiable at $x=a$, then the equation of the tangent line,

$L(x) = f(a) + f'(a)(x-a)$, defines the linearization of f at a .

The approximation $f(x) \approx L(x)$ is the standard linear approximation of f at a . The point $x = a$ is the center of the approximation.

Example1: Find the standard linearization of $\sqrt{1+x}$ at $x=0$ and use the expression to estimate $\sqrt{1.02}$

Example2: Find the standard linearization of $f(x)=\cos x$ at $x=\frac{\pi}{2}$ and use the expression approximate $\cos 1.75$.

Example 3: Approximating Binomial Powers

$$(1 + x)^k \approx 1 + kx$$

Where k is a real number and x is close to 0

Approximate $\frac{1}{\sqrt{1-x^2}}$

Example 4: Linear approximation of radical expressions. Use standard linearization to approximate

a) $\sqrt{123}$

b) $\sqrt[3]{123}$