Notes

CALCULUS 12

# Limits of Rational Functions

1. Sketch a graph of $f(x) = \frac{2x^2+6x}{2x^2+x-15}$

- Factor (if possible) and simplify.

$$\frac{2x^2+6x}{2x^2+x-15} = \frac{2x\cancel{(x+3)}}{(2x-5)\cancel{(x+3)}} = \frac{2x}{2x-5}$$

- Determine whether a point of discontinuity exists and if it does, state its coordinates.

PD: $x+3=0$
$x=-3$

y-coordinate: $\frac{2(-3)}{2(-3)-5} = \frac{-6}{-11} = \frac{6}{11}$

$\therefore$ A point of discontinuity is $\left(-3, \frac{6}{11}\right)$.

- Determine whether a vertical asymptote exists and if it does, state its equations.

VA: $2x-5=0$
$\frac{2x}{2} = \frac{5}{2}$
$x = \frac{5}{2}$

$\therefore$ There is a vertical asymptote at $x=2.5$.

- Rewrite the equation in the form $f(x) = \frac{\pm a}{x-h} + k$ and describe how $f(x)$ relates to $\frac{1}{x}$.

$$\frac{2x}{2x-5} = \frac{2x-5+5}{2x-5} = \frac{2x-5}{2x-5} + \frac{5}{2x-5} = 1 + \frac{5}{2x-5}$$
$$= \frac{5}{2(x-2.5)} + 1$$

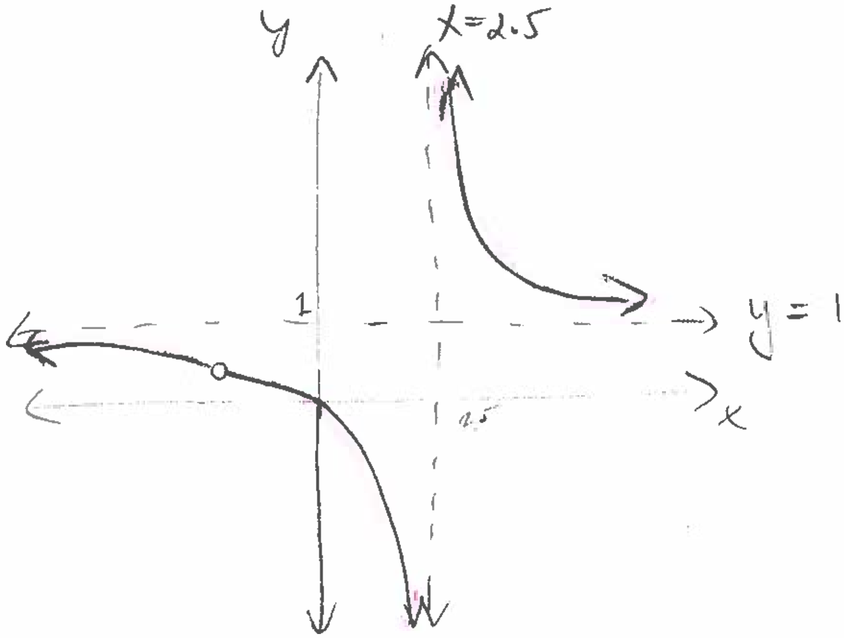
- Determine whether a horizontal asymptote exists and if it does, state its equation.

HA $\rightarrow$ given by vertical displacement $\therefore$ HA: $\underline{y=1}$

- Classify all discontinuities if the function has them.

2 discontinuities — non-removable at VA = infinite
— removable at PD $\left(-3, \frac{6}{11}\right)$.

- Sketch the graph without using graphing technology.




- Determine the following limits:

| |
|---|
| $\lim_{x \to -4} \left( \dfrac{2x^2 + 6x}{2x^2 + x - 15} \right) = \dfrac{2(-4)^2 + 6(-4)}{2(-4)^2 - 4 - 15} = \dfrac{8}{13}$ <br> $\doteq 0.615$ |
| $\lim_{x \to 0^-} \left( \dfrac{2x^2 + 6x}{2x^2 + x - 15} \right) = \dfrac{0}{-15} = 0$ |
| $\lim_{x \to -\infty} \left( \dfrac{2x^2 + 6x}{2x^2 + x - 15} \right) = \lim_{x \to -\infty} \dfrac{2x^2}{2x^2} = 1$ <br> ↑ <br> HA |

2. Sketch a graph of $f(x)=\frac{x^2+5x+4}{x^2+x-12}$

- Factor (if possible) and simplify.

$$\frac{(x+1)\cancel{(x+4)}}{\cancel{(x+4)}(x-3)} \rightarrow f(x)=\frac{x+1}{x-3}$$

- Determine whether a point of discontinuity exists and if it does, state its coordinates.

PD at $x+4=0$
$x=-4$

$\therefore$ PD at $(-4, 3/7)$

y-coordinate $\frac{(-4)+1}{(-4)-3}=\frac{-3}{-7}=\frac{3}{7}$

- Determine whether a vertical asymptote exists and if it does, state its equations.

VA: $x-3=0$
$x=3$

$\therefore$ VA at $x=3$.

- Rewrite the equation in the form $f(x)=\frac{\pm a}{x-h}+k$ and describe how $f(x)$ relates to $\frac{1}{x}$.

$$\frac{x+1}{x-3}=\frac{x-3+4}{x-3}=\frac{x-3}{x-3}+\frac{4}{x-3}=\frac{4}{x-3}+1$$

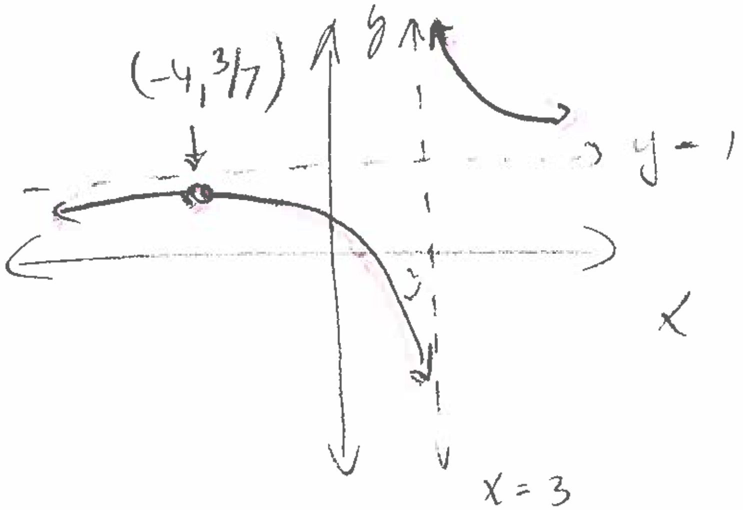
- Determine whether a horizontal asymptote exists and if it does, state its equation.

HA: $y=1$ OR $\lim_{x\to\infty}\frac{x^2+5x+4}{x^2+x-12}=\lim_{x\to\infty}\frac{x^2}{x^2}=\boxed{1}$

- Classify all discontinuities if the function has them.

- a removable discontinuity at $(-4, 3/7)$
- a non-removable discontinuity at $x=3$.
(infinite)

- Sketch the graph without using graphing technology.




- Determine the following limits:

| |
|---|
| $\lim_{x\to -4}\left(\frac{x^2+5x+4}{x^2+x-12}\right) = 3/7$ |
| $\lim_{x\to 0^-}\left(\frac{x^2+5x+4}{x^2+x-12}\right) = \frac{0^2+5(0)+4}{0^2+0-12} = \frac{4}{-12} = \underline{\underline{-\frac{1}{3}}}$ |
| $\lim_{x\to -\infty}\left(\frac{x^2+5x+4}{x^2+x-12}\right) = 1$ <br> HA <br> * $\lim_{x\to\infty} \frac{x^2+5x+4}{x^2+x-12}$ |

$$= \lim_{x\to -\infty} \frac{\cancel{x^2}}{\cancel{x^2}} = \lim_{x\to -\infty} 1 = \underline{\underline{1}}$$

end-behaviour model

$f(x) = -2x + 8$

3. Sketch a graph of $f(x) = \frac{x^2+8x-48}{-6-0.5x}$ $= \frac{x^2+8x-48}{-0.5x-6}$

- Factor (if possible) and simplify.

$\frac{(\cancel{x+12})(x-4)}{-0.5(\cancel{x+12})} = \frac{x-4}{-0.5} = -\frac{x-4}{0.5} = \frac{-(x-4)}{0.5} = \frac{-x}{0.5} + \frac{4}{0.5}$

- Determine whether a point of discontinuity exists and if it does, state its coordinates.

· PD at $x+12=0$
$x = -12$

y-coordinate:
$-2(-12)+8 = 24+8 = 32$
$\therefore$ PD at $(-12, 32)$.

- Determine whether a vertical asymptote exists and if it does, state its equations.

no VA as a simplified f(x) has no NPVs!

- Rewrite the equation in the form $f(x) = \frac{\pm a}{x-h} + k$ and describe how $f(x)$ relates to $\frac{1}{x}$.

→ N/A as there is no x in the denominator

→ f(x) is linear with a PD.

- Determine whether a horizontal asymptote exists and if it does, state its equation.

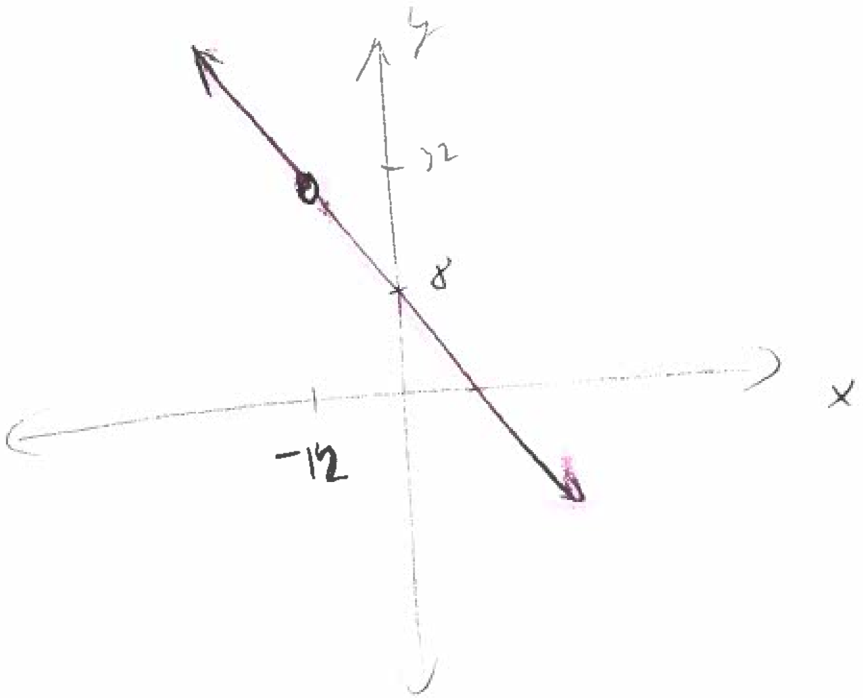
→ no HA as $\lim_{x \to \pm\infty} f(x) = \mp\infty$ not a $\mathbb{R}$ #

- Classify all discontinuities if the function has them.

- only 1 discontinuity:

- PD = removable at $(-12, 32)$

- Sketch the graph without using graphing technology.




- Determine the following limits:

| |
|---|
| $\lim_{x\to -12} \frac{x^2+8x-48}{-6-0.5x} = \lim_{x\to -12} -2x+8 = (-2)(-12)+8 = \underline{\underline{32}}$ |
| $\lim_{x\to 0^-} \frac{x^2+8x-48}{-6-0.5x} = \frac{0^2+0-48}{-6-0.5(0)} = \frac{-48}{-6} = \underline{\underline{8}}$ |
| $\lim_{x\to -\infty} \frac{x^2+8x-48}{-6-0.5x} = +\infty$ <br> end behaviour |