Limits of Rational Functions

1. Sketch a graph of $f(x) = \frac{2x^2+6x}{2x^2+x-15}$
   - Factor (if possible) and simplify.

   - Determine whether a point of discontinuity exists and if it does, state its coordinates.

   - Determine whether a vertical asymptote exists and if it does, state its equations.

   - Rewrite the equation in the form $f(x) = \frac{\pm a}{x-h} + k$ and describe how $f(x)$ relates to $\frac{1}{x}$.

   - Determine whether a horizontal asymptote exists and if it does, state its equation.

   - Classify all discontinuities if the function has them.

- Sketch the graph without using graphing technology.

- Determine the following limits:

$$\lim_{x \to -4} \left(\frac{2x^2 + 6x}{2x^2 + x - 15} \right)$$
$$\lim_{x \to 0^-} \left(\frac{2x^2 + 6x}{2x^2 + x - 15} \right)$$
$$\lim_{x \to -\infty} \left(\frac{2x^2 + 6x}{2x^2 + x - 15} \right)$$

2. Sketch a graph of $f(x) = \frac{x^2+5x+4}{x^2+x-12}$
   - Factor (if possible) and simplify.

   - Determine whether a point of discontinuity exists and if it does, state its coordinates.

   - Determine whether a vertical asymptote exists and if it does, state its equations.

   - Rewrite the equation in the form $f(x) = \frac{\pm a}{x-h} + k$ and describe how $f(x)$ relates to $\frac{1}{x}$.

   - Determine whether a horizontal asymptote exists and if it does, state its equation.

   - Classify all discontinuities if the function has them.

- Sketch the graph without using graphing technology.

- Determine the following limits:

$$\lim_{x \to -4} \left(\frac{x^2 + 5x + 4}{x^2 + x - 12} \right)$$

$$\lim_{x \to 0^-} \left(\frac{x^2 + 5x + 4}{x^2 + x - 12} \right)$$

$$\lim_{x \to -\infty} \left(\frac{x^2 + 5x + 4}{x^2 + x - 12} \right)$$

3. Sketch a graph of $f(x) = \dfrac{x^2+8x-48}{-6-0.5x}$
   - Factor (if possible) and simplify.

   - Determine whether a point of discontinuity exists and if it does, state its coordinates.

   - Determine whether a vertical asymptote exists and if it does, state its equations.

   - Rewrite the equation in the form $f(x) = \dfrac{\pm a}{x-h} + k$ and describe how $f(x)$ relates to $\dfrac{1}{x}$.

   - Determine whether a horizontal asymptote exists and if it does, state its equation.

   - Classify all discontinuities if the function has them.

- Sketch the graph without using graphing technology.

- Determine the following limits:

$\lim\limits_{x \to -12} \dfrac{x^2 + 8x - 48}{-6 - 0.5x}$
$\lim\limits_{x \to 0^-} \dfrac{x^2 + 8x - 48}{-6 - 0.5x}$
$\lim\limits_{x \to -\infty} \dfrac{x^2 + 8x - 48}{-6 - 0.5x}$