CALCULUS 12 LIMITS AND THEIR PROPERTIES

"I told Brad I was approaching 30...I just didn't say from which direction."

One-Sided Limits

Right-hand limit:

$$\lim_{x \to c^+} f(x) = L$$

"A limit of f(x) as x approaches c from the right"

Left-hand limit:

$$\lim_{x \to c^-} f(x) = L$$

"A limit of f(x) as x approaches c from the left"

THEOREM:

A function f(x) has a limit as x approaches c <u>if and only if</u> the right-hand and left-hand limits at c are equal.

$$\lim_{x \to c} f(x) = L \Leftrightarrow \lim_{x \to c^+} f(x) = L \text{ and } \lim_{x \to c^-} f(x) = L$$

$\lim_{x \to c} f(x) = L$ can be referred to as a two-sided limit

A function f has a limit L as x approaches c if, given any positive number ε, there exists a positive numberδ such that for all x

$$0 < |x - c| < \delta \Rightarrow |f(x) - L| < \varepsilon$$

Note: A function can have a limit as x approaches c even when the function is not defined at c or is discontinuous at c.

Properties of limits:

1. Limit of a function that has a constant value is equal to this value.

Ex. f(x) = 5

Ex. g(x)= -7

In general:

$$\lim_{x \to c}(k) = k$$

2. Limit of the identity function f(x)=x when x approaches is equal to c.

In general,

$$\lim_{x \to c}(x) = c$$

If L, M, c, and k are real numbers and $\lim_{x \to c} f(x) = L$ **and** $\lim_{x \to c} g(x) = M$, **then**

3. The limit of the sum of two functions is the sum of their limits. Sum Rule.

4. The limit of the difference of two functions is the difference of their limits. Difference Rule.

5. The limit of a product of two functions is the product of their limits. Product Rule.

6. The limit of a constant times a function is the constant times the limit of the function. Constant Multiple Rule.

7. The limit of a quotient of two functions is the quotient of their limits, provided the limit of the denominator is not zero. Quotient Rule.

8. The limit of a rational power of a function is that power of the limit of the function , provided the latter is a real number. Power Rule.

THEOREM: Limits of Polynomial and Rational Functions

1. If $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_0$ is any polynomial function and c is any real number, then

$$\lim_{x \to c} f(x) = f(c) = a_n c^n + a_{n-1} c^{n-1} + \cdots + a_0$$

2. If f(x) and g(x) and polynomials and c is any real number, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{f(c)}{g(c)}, provided\ thet\ g(c) \neq 0$$

Assignment:

p.65-68 #1-58

KNOW YOUR LIMITS!

$$\lim\,[f(x) + g(x)] = \lim f(x) + \lim g(x)$$

$$\lim\,[f(x) \cdot g(x)] = \lim f(x) \cdot \lim g(x)$$

$$\lim\,[f(x) - g(x)] = \lim f(x) - \lim g(x)$$

$$\lim \frac{f(x)}{g(x)} = \frac{\lim f(x)}{\lim g(x)}$$

if $\lim g(x) \neq 0$

$$\lim_{x \to a} f(x) = f(a)$$

if f is continuous at a

$$\lim c\,f(x) = c \lim f(x)$$

$$\lim_{x \to a} c = c$$

$$\lim f(g(x)) = f(\lim g(x))$$

$$\lim_{x \to a} x = a$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Definition of derivative of $f(x)$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}$$

L'Hôpital's Rule

$$\lim_{x \to 0} (1 + x)^{1/x} = e$$

Definition of e