CALCULUS 12 **LIMITS AND CONTINUITY ASSIGNMENT** Name:_____

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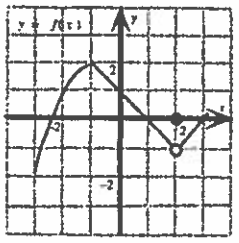
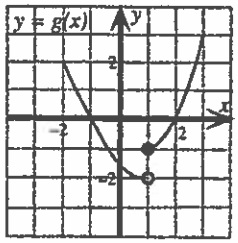
Use the graphs to find each limit, if it exists.





1. $\lim_{x\to 2}\big(f(x)+g(x)\big)$
2. $\lim_{x\to -1}\dfrac{f(x)}{g(x)}$
3. $\lim_{x\to 1}\sqrt{3+f(x)}$
4. $\lim_{x\to 2}x^3 f(x)$
5. $\lim_{x\to 1^-}g(x)$
6. $\lim_{x\to 2^+}\dfrac{f(x)}{g(x)}$
7. $\lim_{x\to 2}f\big(g(x)\big)$
8. $\lim_{x\to -1}\big(2f(x)g(x)\big)$

9. Find the value(s) of x for which $h(x)=\dfrac{2x+1}{2x^2+11x+5}$ is discontinuous and identify these discontinuities as removable or nonremovable.

10. Draw a function with each of the following characteristics:
    a) point discontinuity
    b) infinite discontinuity
    c) jump discontinuity
    d) oscillating discontinuity

11. Write the definition of continuity at a point.

Find each limit, if it exists.

12. $\lim_{x\to 0}\dfrac{\sin 3x}{x}$
13. $\lim_{x\to 2}\sec\dfrac{\pi x}{3}$
14. $\lim_{x\to 0}\dfrac{x+\sin x}{x}$
15. $\lim_{x\to 3^-}\sqrt{9-x^2}$
16. $\lim_{x\to\infty}\dfrac{2x}{\sqrt{x^2+1}}$
17. $\lim_{x\to 2}\dfrac{x^3-1}{x-2}$
18. $\lim_{x\to 2}\dfrac{x^2-4}{x^2+4}$
19. $\lim_{h\to 0}\dfrac{(3+h)^2-9}{h}$
20. $\lim_{x\to 1}\left(5-\dfrac{2}{(x-1)^2}\right)$
21. $\lim_{x\to 0}\dfrac{\frac{1}{2+x}-\frac{1}{2}}{x}$
22. $\lim_{x\to 0}f(x)$, if $f(x)=\begin{cases}\dfrac{|x|}{x}, & x\neq 0\\ 0, & x=0\end{cases}$
23. $\lim_{x\to -2}(x-6)^{2/3}$
24. $\lim_{x\to \pi^-}\dfrac{\sin x}{1-\cos x}$
25. $\lim_{x\to -\infty}\dfrac{5x^3-2x+7}{3x^3+2x^2}$

For the next three questions, use $f(x)=\begin{cases}3-x, & x<2\\ \dfrac{x}{2}+1, & x>2\end{cases}$.

26. $\lim_{x\to 2^+}f(x)$
27. $\lim_{x\to 2^-}f(x)$
28. $\lim_{x\to 2}f(x)$

For the next three questions, use $f(x)=\begin{cases}3-x, & x<2\\ 2, & x=2.\\ \frac{x}{2}, & x>2\end{cases}$

29. $\lim_{x\to 2^-} f(x)$

30. $\lim_{x\to 2^+} f(x)$

31. $\lim_{x\to 2} f(x)$

For the next three questions, let $f(x)=\dfrac{x^2-3x-10}{x^2-4}$.

32. Find $\lim_{x\to -2} f(x)$, if it exists.

33. Sketch a graph of $f(x)$.

34. Find a new function, $g(x)$, that agrees with $f(x)$ at all but one point and is defined at $x=-2$.

35. Determine the value of c such that each function is continuous on $(-\infty, \infty)$.

a) $f(x)=\begin{cases}3x-2, & x<5\\ x^2+c, & x\geq 5\end{cases}$

b) $g(x)=\begin{cases}x^2-1, & x<3\\ 2cx, & x\geq 3\end{cases}$

36. Find all vertical and horizontal asymptotes of each function.

a) $g(x)=\dfrac{x-2}{2x^2+3x-5}$ b) $f(x)=\dfrac{x^2-2x}{x+2}$ c) $h(x)=\dfrac{x^2-4}{x^2+4}$

37. In your own words, explain the Intermediate Value Theorem. Include a sketch with your answer.

38. Identify all asymptotes and discontinuities of the graph of $y=\dfrac{3x^2-x+5}{x^2-4}$.

For the next three questions, let $f(x)=\begin{cases}\frac{x^2+x}{x}, & x\neq 0\\ 0, & x=0\end{cases}$.

True or false. Justify your answers.

39. $\lim_{x\to 0} f(x)$ exists.

40. $f(0)$ exists.

41. f is continuous at $x=0$.

42. Let $g(x)=4x^2-x$.
   a) Find the average rate of change of g on $[0, 5]$.
   b) Find the rate of change of g at $x=2$.
   c) Find the equation of the tangent line at $x=2$.
   d) Find the equation of the normal line at $x=2$.