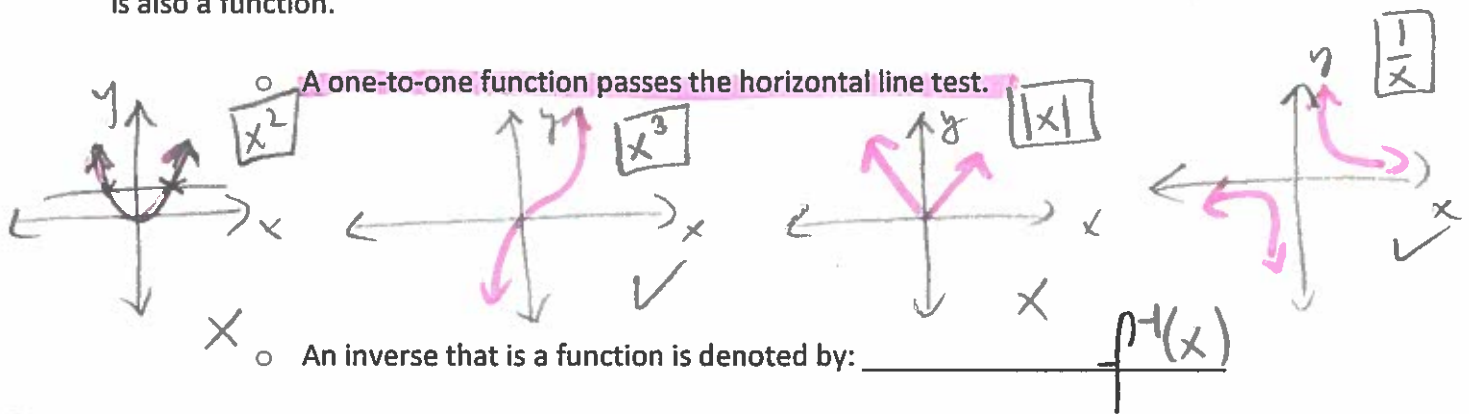


Notes:

Inverse of a Relation

- Mapping notation: $(x, y) \rightarrow (y, x)$
This means that the domain of the original becomes the range of the inverse and the range of the original becomes the domain of the inverse.
- Every relation has an inverse. Inverse of a relation is a "special reflection" of the original relation where the mirror of the reflection is the line $y = x$.
- When the inverse transformation is carried out on a function that is **one-to-one** then the inverse itself is also a function.



- Despite the notation an inverse is very different from a reciprocal.
- It is possible to strategically restrict the domain of the original function so it becomes one-to-one and its inverse is then also a function. This is most commonly done with trigonometric functions.
- In general, the equation of an inverse can be found algebraically by following these steps:
 1. Replace $f(x)$ with "y".
 2. Swap every "x" with "y" and "y" with "x".
 3. Solve for "y".
 4. Use the inverse notation if the resultant relation is also a function.

Example 1: Find the inverse of $f(x) = 0.25x - 5$

$$\begin{aligned}
 y &= 0.25x - 5 \\
 x &= 0.25y - 5 \\
 x + 5 &= 0.25y \\
 \frac{x + 5}{0.25} &= \frac{0.25y}{0.25}
 \end{aligned}$$

$$4x + 20 = y$$

$$\therefore f^{-1}(x) = 4x + 20$$

$$f^{-1}(x) = \frac{1}{m}x - \frac{b}{m}$$

Notice: Slopes are reciprocals.

* last page

Example 2: Find the inverse of $f(x) = 2x^2 + 16x - 5$. Sketch a graph of the original and of the inverse in the same coordinate system.

$$y = 2x^2 + 16x - 5$$

$$x = 2y^2 + 16y - 5 \rightarrow \text{complete the square}$$

$$x = 2[y^2 + 8y] - 5$$

$$x = 2[(y+4)^2 - 16] - 5$$

$$x = 2(y+4)^2 - 37$$



not one-to-one

$$\frac{x+37}{2} = \frac{2(y+4)^2}{2}$$

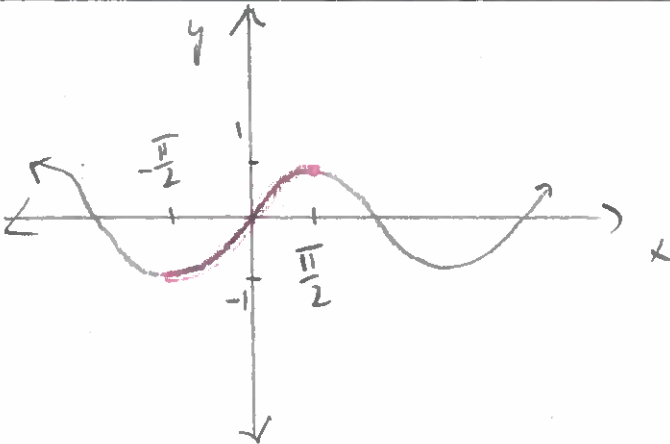
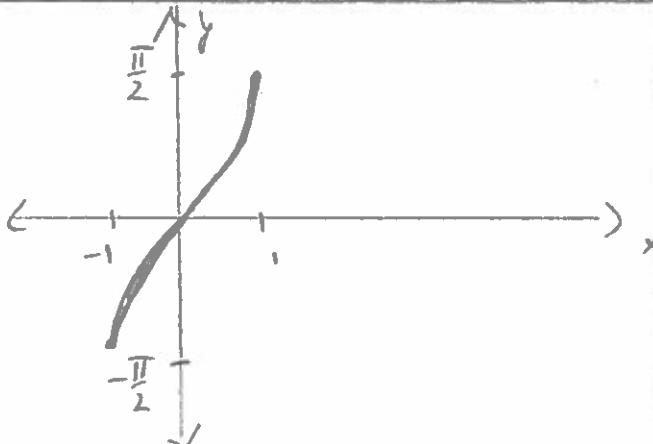
$$\sqrt{\frac{x+37}{2}} = \sqrt{(y+4)^2}$$

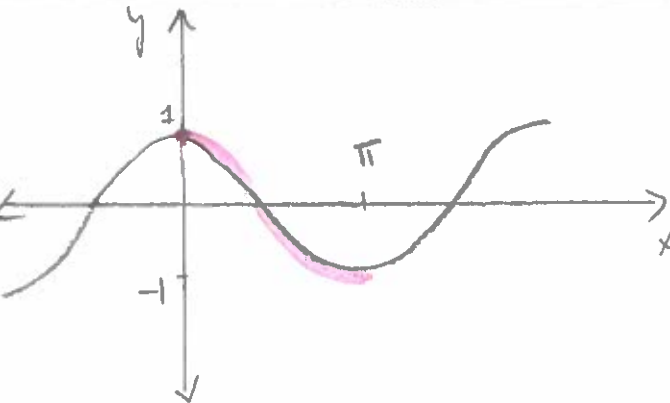
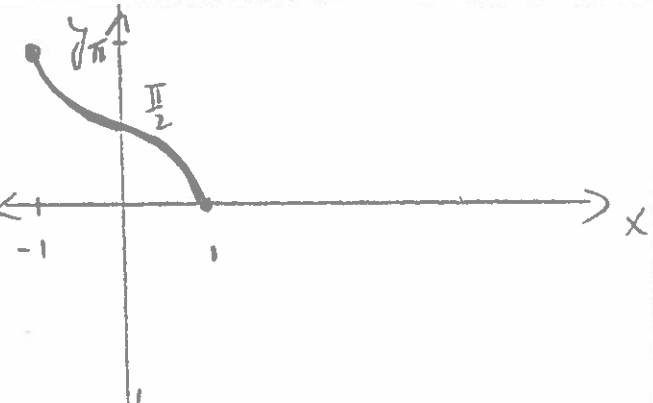
$$\pm \sqrt{\frac{x+37}{2}} - 4 = y \leftarrow \text{not a function}$$

\therefore the inverse of $f(x)$ is $y = \pm \sqrt{\frac{x+37}{2}} - 4$

Inverse of Trigonometric Functions

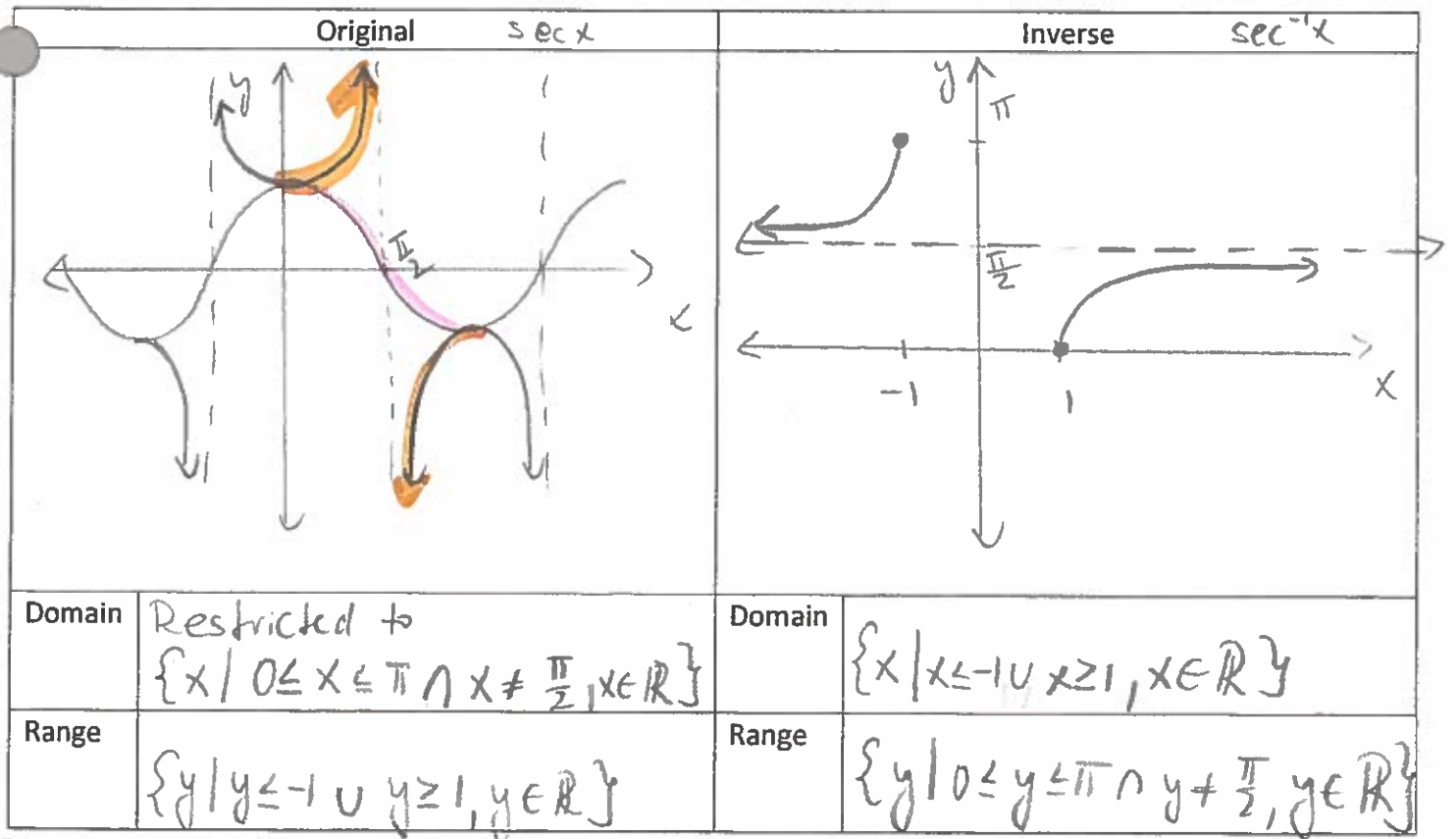
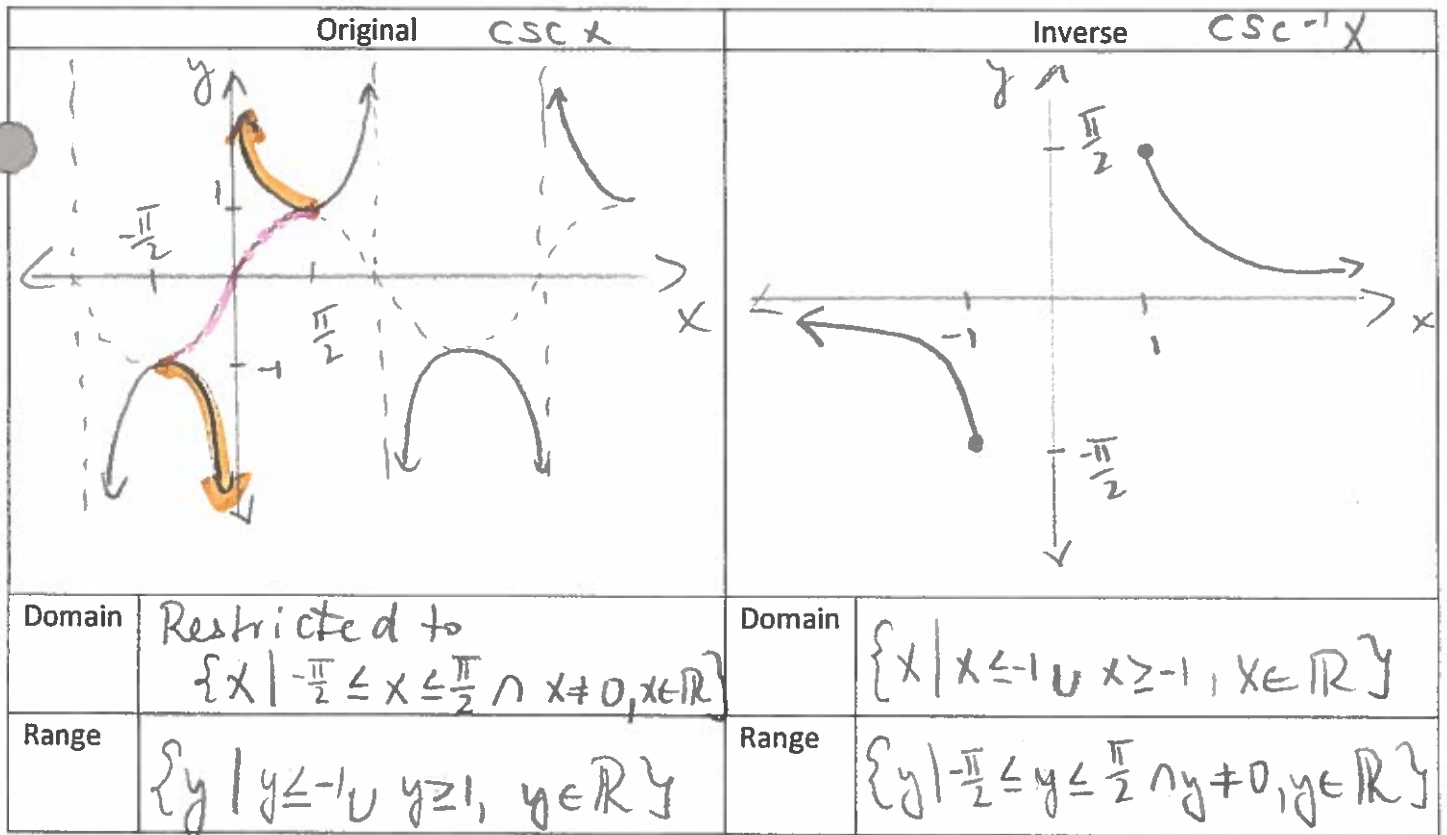
- As all trigonometric functions are periodic, they are not one-to-one.
- In order for the inverse any trigonometric function to be a function, we restrict the domain of the original in a specific manner.

Original $\sin x$		Inverse $\sin^{-1} x$	
			
<u>Domain</u>	Restricted to $\{x \mid -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}, x \in \mathbb{R}\}$	<u>Domain</u>	$\{x \mid -1 \leq x \leq 1, x \in \mathbb{R}\}$
<u>Range</u>	$\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$	<u>Range</u>	$\{y \mid -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, y \in \mathbb{R}\}$

Original $\cos x$		Inverse $\cos^{-1} x$	
			
<u>Domain</u>	Restricted to $\{x \mid 0 \leq x \leq \pi, x \in \mathbb{R}\}$	<u>Domain</u>	$\{x \mid -1 \leq x \leq 1, x \in \mathbb{R}\}$
<u>Range</u>	$\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$	<u>Range</u>	$\{y \mid 0 \leq y \leq \pi, y \in \mathbb{R}\}$

Original $\tan x$		Inverse $\tan^{-1} x$	
Domain	Restricted to $\{x \mid -\frac{\pi}{2} < x < \frac{\pi}{2}, x \in \mathbb{R}\}$	Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \in \mathbb{R}\}$	Range	$\{y \mid -\frac{\pi}{2} < y < \frac{\pi}{2}, y \in \mathbb{R}\}$

Original $\cot x$		Inverse $\cot^{-1} x$	
Domain	Restricted to $\{x \mid 0 < x < \pi, x \in \mathbb{R}\}$	Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \in \mathbb{R}\}$	Range	$\{y \mid 0 < y < \pi, y \in \mathbb{R}\}$



Example 2 Sketch:

