Implicit Differentiation

- this method is used when a function is given as an expression in two variables.
- think about y as a function of x

Example 1: Find $\frac{dy}{dx}$ for a circle with radius $r=5$.

$x^2+y^2=25 \qquad (x^2+y^2=r^2)$

$2x+2y\cdot y'=0$

$2yy'=-2x$

$y'=\frac{-2x}{2y}$

$y'=\frac{-x}{y}$

$\therefore \frac{dy}{dx}=\frac{-x}{y}$.

Example 2: Find $\frac{dy}{dx}$.

$x^3 + y^3 = 18x \cdot y$ ("xy" is a product of two functions:
- the linear function 'x'
- and the "mysterious" function "y".

$$3x^2 + 3y^2 \cdot y' = 18(1)y + 18xy'$$

$$-18xy' + 3y^2y' = 18y - 3x^2$$

$$y'(3y^2 - 18x) = 18y - 3x^2$$

$$y' = \frac{18y - 3x^2}{3y^2 - 18x}$$

$$\therefore \quad y' = \frac{6y - x^2}{y^2 - 6x}$$

Example 3: Find dy/dx:

$$x^2 = \frac{x-y}{x+y}$$

$$2x = \frac{(1-y')(x+y) - [(x-y)(1+y')]}{(x+y)^2}$$

$$2x(x+y)^2 = \cancel{x} + y - xy' - \cancel{yy'} - [\cancel{x} + xy' - y - \cancel{yy'}]$$

$$2x(x+y)^2 = 2y - 2xy'$$

$$\frac{2x(x+y)^2 - 2y}{-2x} = \frac{-2xy'}{-2x}$$

$$y' = \frac{2y - 2x(x+y)^2}{2x}$$

$$\therefore y' = \frac{y - x(x+y)^2}{x} \quad \text{OR} \quad \frac{y}{x} - (x+y)^2$$

Example 4: Find y'.

$$x^2+4xy+4y^2-3x=6$$

$$2x+4y+4xy'+8yy'-3=0$$

$$4xy'+8yy'=3-2x-4y$$

$$y'(4x+8y)=3-2x-4y$$

$$\therefore y'=\frac{3-2x-4y}{4x+8y}=\frac{-2x-4y+3}{4(x+2y)}$$

Example 5: Find y'.

$$x=\cos y$$

$$1=-\sin y\cdot y'$$

$$\therefore y'=\frac{-1}{\sin y} \quad \text{where } y\neq n\pi;\ n\in\mathbb{Z}$$