Notes:

Implicit Differentiation

- Implicit differentiation allows to find derivatives of functions that are not defined explicitly as a function of a single variable

Steps of implicit differentiation:

1. **Treat y as a differentiable function of x.** Differentiate both sides of the equation with respect to x.
2. Collect the terms with dy/dx on one side of the equation.
3. Factor out dy/dx
4. Solve for dy/dx

Examples: Find $\frac{dy}{dx}$ when $x^2 - xy' + y^2 = 7$

$$2x - [(1)(y) + x(1)y^{1-1} \cdot y'] + 2y^{2-1} \cdot y' = 0$$

$$2x - y - xy' + 2y \cdot y' = 0$$

$$-xy' + 2yy' = y - 2x$$

$$\frac{y'(-x+2y)}{(-x+2y)} = \frac{y-2x}{(-x+2y)}$$

$$\boxed{y' = \frac{y-2x}{2y-x}}$$

- **Finding second derivative implicitly**

Example:

second derivative : $\frac{d^2y}{dx^2}$

$$2x^3 - 3y^2 = 8$$

$$6x^2 - [3 \cdot (2)y^{2-1} \cdot y'] = 0$$

$$6x^2 - 6y \cdot y' = 0$$

$$\frac{6x^2}{6y} = \frac{6y \cdot y'}{6y}$$

$$y' = \frac{x^2}{y}$$

this is $\frac{dy}{dx}$

HW:p 153 and p162 Complete as many exercises as you need to feel comfortable using the Chain Rule and the Implicit Differentiation.

$$\frac{d}{dx}\left[y'\right] = \frac{d^2y}{dx^2}$$

So $$\frac{d}{dx}\left[\frac{x^2}{y}\right] = \frac{d}{dx}\left[x^2 \cdot y^{-1}\right]$$

$$y'' = 2x \cdot y^{-1} + x^2 \cdot (-1) y^{-1-1} \cdot y'$$

$$y'' = \frac{2x}{y} - \frac{x^2 \cdot y'}{y^2}$$ ← substitute $y' = \frac{x^2}{y}$

$$y'' = \frac{2x}{y} - \frac{x^2 \cdot \frac{x^2}{y}}{y^2}$$

$$y'' = \frac{2x}{y} - \frac{\frac{x^4}{y}}{y^2}$$

$$\boxed{y'' = \frac{2x}{y} - \frac{x^4}{y^3}} \quad \text{OR} \quad \frac{2xy^2 - x^4}{y^3}$$

provided $y \neq 0$