Notes :

**CALCULUS 12** **FUNCTIONS**

## FORMAL DEFINITION OF INCREASING AND DECREASING FUNCTIONS

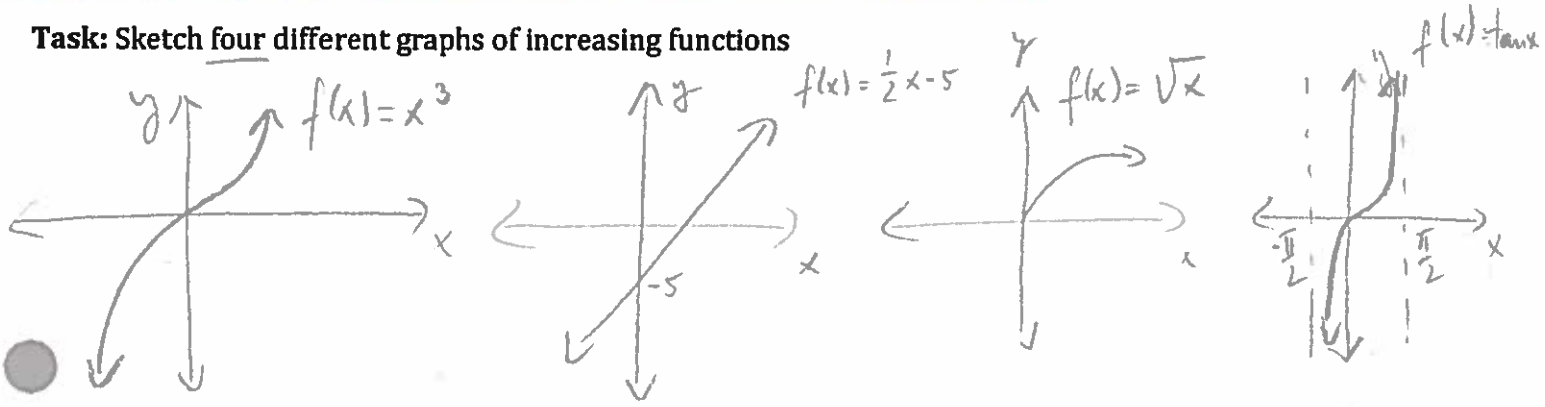
Let f be a function defined on interval I and let $x_1$ and $x_2$ be any two points in I.

1. If $f(x_2) > f(x_1)$ whenever $x_1 < x_2$, then f is said to be **increasing on I.**

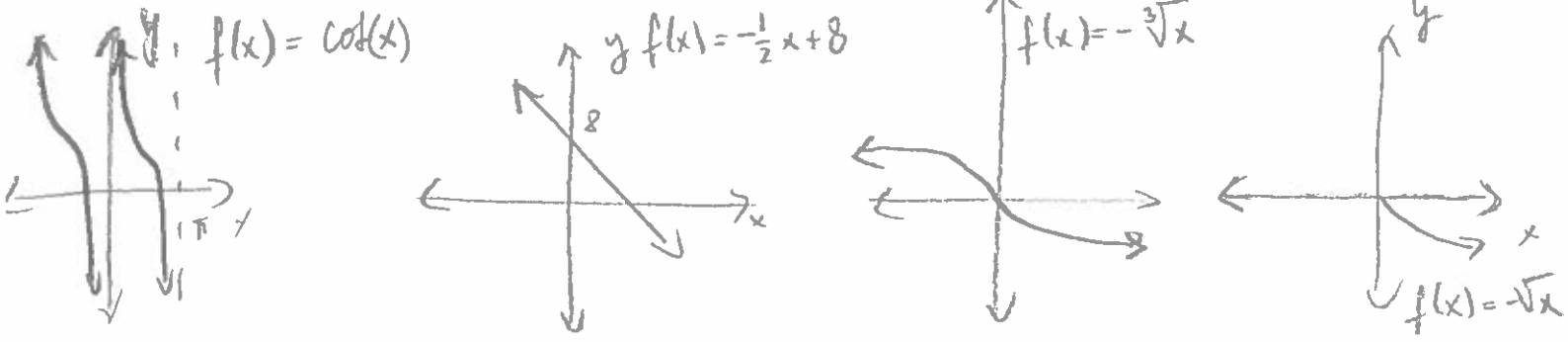
2. If $f(x_2) < f(x_1)$ whenever $x_1 < x_2$, then f is said to be **decreasing on I.**

A function that is increasing or decreasing on I is called MONOTONIC on I.

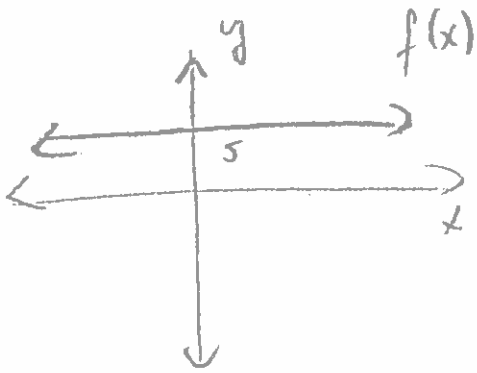
**Task:** Sketch four different graphs of increasing functions




**Task:** Sketch four different graphs of decreasing functions




**Task:** Sketch a graph of a function that is neither increasing nor decreasing over the entire domain of the function.

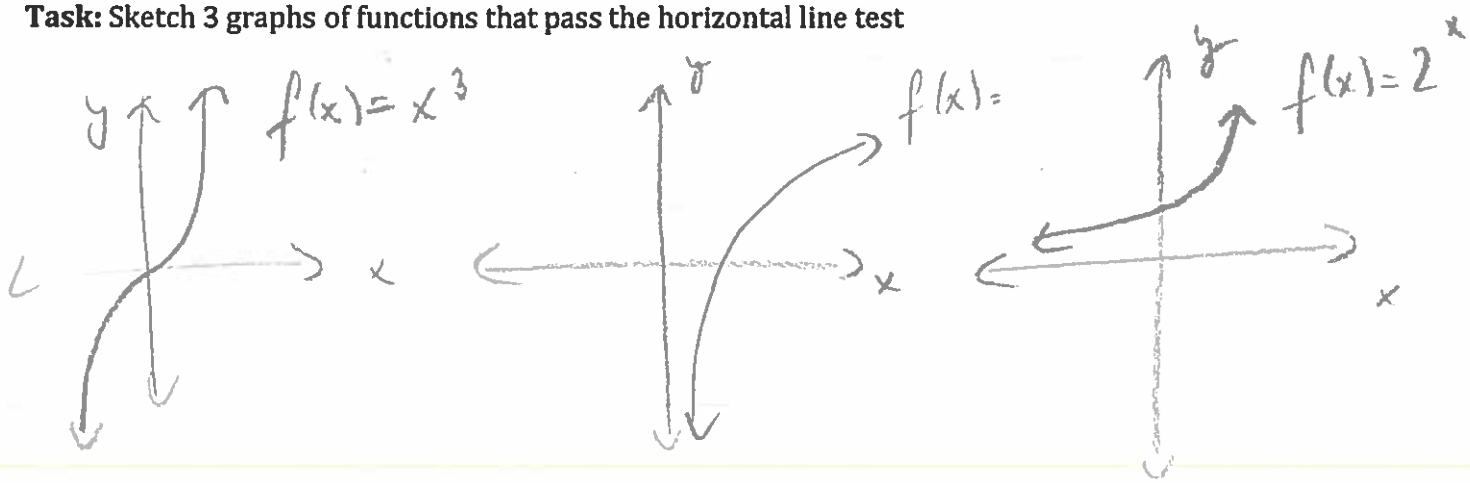



Constant function.

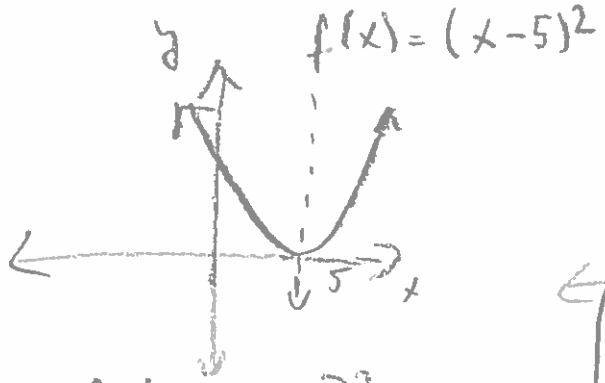
## INVERSE OF A FUNCTION

- For an inverse of a function to be also a function, the original function must pass the **horizontal line test**
- A Function that passes the horizontal line test is called **one-to-one function**

**Task:** Sketch 3 graphs of functions that pass the horizontal line test

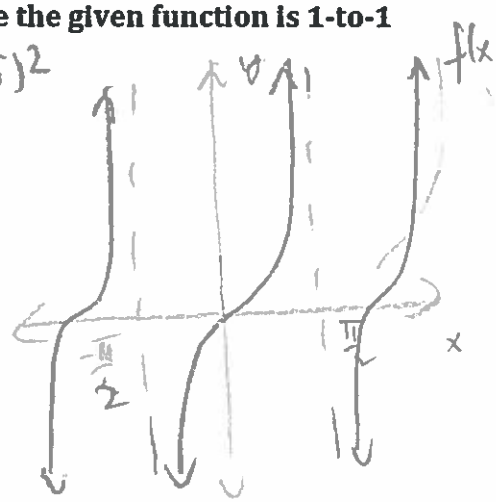



**Task:** Sketch 3 graphs of functions that pass the horizontal line test when their domain is restricted.
**State the interval of the domain where the given function is 1-to-1**

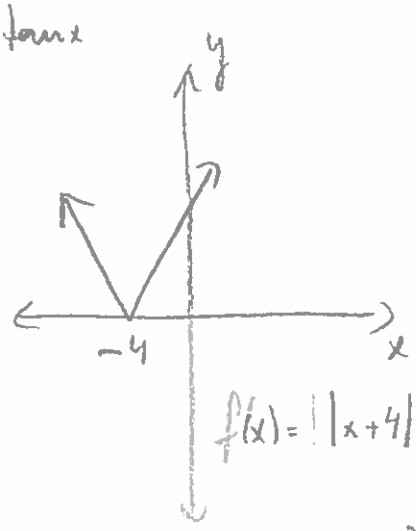



$D_1: \{x \mid x \leq 5, x \in \mathbb{R}\}$

$D_2: \{x \mid x \geq 5, x \in \mathbb{R}\}$




$D: \{x \mid -\frac{\pi}{2} < x < \frac{\pi}{2}, x \in \mathbb{R}\}$




$D_1: \{x \mid x \leq -4, x \in \mathbb{R}\}$

$D_2: \{x \mid x \geq -4, x \in \mathbb{R}\}$

## COMPOSITE FUNCTIONS

**f(g(x)) = "f of g of x" = composing g and f = "g before f" = "first g then f" = f∘g**

**Example: a) Find formula of f(g(x)) if f(x) = x³ +5x and g(x) = 0.5x -2. Then evaluate f(g(x)) at x=1**

$f(x) = x^3 + 5x$

$g(x) = 0.5x - 2$

$f(g(x)) = [g(x)]^3 + 5[g(x)]$

$= (0.5x-2)^3 + 5(0.5x-2)$

$= (0.25x^2 - 2x + 4)(0.5x-2) + 2.5x - 10$

$= 0.125x^3 - 1.5x^2 + 6x - 8 + 2.5x - 10$

$f(g(x)) = 0.125x^3 - 1.5x^2 + 8.5x - 18$

$f(g(1)) = 0.125 - 1.5 + 8.5 - 18$

$= -10.875$

**b) Find formula of g(f(x)) and evaluate it at x=5**

$g(f(x)) = 0.5[f(x)] - 2$

$= 0.5(x^3 + 5x) - 2$

$= 0.5x^3 + 2.5x - 2$

$g(f(5)) = 0.5(5)^3 + 2.5(5) - 2$

$= 73$

**HW: p 21 #63-66**