

KEY

Factoring: All Techniques Combined (Hard)

Date _____

Period _____

Factor each.

$$\begin{aligned}
 1) \quad & x^3 - 5x^2 - x + 5 \\
 & = x^2(x-5) - 1(x-5) \\
 & = (x-5)(x^2-1) \\
 & = \underline{(x-5)(x-1)(x+1)}
 \end{aligned}$$

$$\begin{aligned}
 3) \quad & x^6 - 26x^3 - 27 \quad \text{let } A = x^3 \\
 & A^2 - 26A - 27 \\
 & = (A-27)(A+1) \\
 & = (x^3-27)(x^3+1) \leftarrow \text{cubes}
 \end{aligned}$$

$$= \underline{(x-3)(x^2+3x+9)(x+1)(x^2-x+1)}$$

$$5) \quad x^4 - 13x^2 + 40$$

$$\begin{aligned}
 \text{let } & A = x^2 \\
 & A^2 - 13A + 40 \\
 & (A-8)(A-5) \\
 & = \underline{(x^2-8)(x^2-5)}
 \end{aligned}$$

$$\begin{aligned}
 7) \quad & x^6 - 4x^2 = x^2(x^4-4) \\
 & = \underline{x^2(x^2-2)(x^2+2)}
 \end{aligned}$$

OR

$$= x^2(x-\sqrt{2})(x+\sqrt{2})(x^2+2)$$

$$\begin{aligned}
 2) \quad & x^4 - 2x^2 - 15 = \underline{A^2 - 2A - 15} \\
 & = (x^2-5)(x^2+3)
 \end{aligned}$$

$$\begin{aligned}
 & \text{Substitute back } A = x^2 \\
 & \quad \quad \quad (A-5)(A+3)
 \end{aligned}$$

$$\begin{aligned}
 4) \quad & x^6 + 2x^4 - 16x^2 - 32 \\
 & = x^4(x^2+2) - 16(x^2+2) \\
 & = (x^2+2)(x^4-16) \\
 & = (x^2+2)(x^2-4)(x^2+4) \\
 & = \underline{(x^2+2)(x^2+4)(x-2)(x+2)}
 \end{aligned}$$

$$6) \quad x^9 - x^6 - x^3 + 1$$

$$\begin{aligned}
 & = x^6(x^3-1) - 1(x^3-1) \\
 & = (x^6-1)(x^3-1) \\
 & = (x^3-1)(x^3+1)(x^3-1) = \underbrace{(x^3-1)^2(x^3+1)}_{\text{cubes}}
 \end{aligned}$$

$$= \underline{(x-1)^2(x^2+x+1)^2(x+1)(x^2-x+1)}$$

$$8) \quad x^4 + 14x^2 + 45$$

$$\begin{aligned}
 & A = x^2 \rightarrow A^2 + 14A + 45 \\
 & = (A+9)(A+5) \\
 & = \underline{(x^2+9)(x^2+5)}
 \end{aligned}$$

$$A = x^2$$

$$\begin{aligned} 9) \quad & 2x^4 + x^2 - 6 \\ & = 2A^2 + A - 6 \\ & = 2A^2 + 4A - 3A - 6 \\ & = 2A(A+2) - 3(A+2) \\ & = \underline{(x^2+2)(2x^2-3)} \end{aligned}$$

$$\begin{aligned} 11) \quad & 4x^3 - x^2 - 4x + 1 \\ & = x^2(4x-1) - 1(4x-1) \\ & = (x^2-1)(4x-1) \\ & = \underline{(x-1)(x+1)(4x-1)} \end{aligned}$$

$$\begin{aligned} 13) \quad & 5x^2 + 24x - 5 \\ & = 5x^2 - x + 25x - 5 \\ & = x(5x-1) + 5(5x-1) \\ & = \underline{(x+5)(5x-1)} \end{aligned}$$

$$\begin{aligned} 15) \quad & 4x^2 + 4x - 15 \\ & = 4x^2 - 6x + 10x - 15 \\ & = 2x(2x-3) + 5(2x-3) \\ & = \underline{(2x+5)(2x-3)} \end{aligned}$$

$$17) \quad \frac{-64x^3}{-1} + \frac{125}{1} = 0$$

$$64x^3 - 125 = 0$$

$$(4x)^3 - 5^3 = 0$$

$$(4x-5)(16x^2 + 20x + 25) = 0$$

$$\begin{aligned} 10) \quad & 2x^2 - 13x + 20 \\ & = 2x^2 - 8x - 5x + 20 \\ & = 2x(x-4) - 5(x-4) \\ & = \underline{(2x-5)(x-4)} \end{aligned}$$

$$\begin{aligned} 12) \quad & 4x^8 - 61x^4 + 225 \\ & = 4x^8 - 25x^4 - 36x^4 + 225 \\ & = x^4(4x^4 - 25) - 9(4x^4 - 25) \\ & = (x^4 - 9)(4x^4 - 25) \\ & = \underline{(x^2-3)(x^2+3)(2x^2-5)(2x^2+5)} \end{aligned}$$

$$\begin{aligned} 14) \quad & 5x^2 + 29x + 20 \\ & = 5x^2 + 4x + 25x + 20 \\ & = x(5x+4) + 5(5x+4) \\ & = \underline{(x+5)(5x+4)} \end{aligned}$$

$$\begin{aligned} 16) \quad & 10x^3 - 8x^2 + 25x - 20 \\ & = 2x^2(5x-4) + 5(x-4) \\ & = \underline{(2x^2+5)(5x-4)} \end{aligned}$$

$$\begin{aligned} 18) \quad & 8x^4 + 10x^2 - 3 \\ & = 8x^4 + 2x^2 + 12x^2 - 3 \\ & = 2x^2(4x^2-1) + 3(4x^2-1) \\ & = (2x^2+3)(4x^2-1) \\ & = \underline{(2x^2+3)(2x-1)(2x+1)} \end{aligned}$$