CALCULUS 12

EXTREME VALUES OF FUNCTIONS

DEFINTION: **ABSOLUTE EXTREME VALUES**

Let f be a function with domain D. Then f(c) is the

1. **absolute maximum value** on D if and only if $f(x) \leq f(c)$ for all x in D.

2. **absolute minimum value** on D if and only if $f(x) \geq f(c)$ for all x in D.

- Absolute = global maximum and minimum values are also called **absolute extrema**

Function	Domain	Graph	Absolute Extrema on D	
			Minimum	Maximum

Theorem: **The Extreme Value Theorem**

If f is continuous on a closed interval [a,b], then f has both maximum value and a minimum value on the interval.

Maximum and minimum at interior points	Maximum and minimum at exterior points	Maximum at an interior point and minimum at an exterior point	Minimum at an interior point and maximum at an exterior point

Local = relative extreme values

DEFINITION: **LOCAL EXTREME VALUES**

Let c be an interior point of the domain of the function f. Then f(c) is a

1. **local maximum value** at c if and only if $f(x) \leq f(c)$ for all x in some open interval containing c.

2. **local minimum value** at c if and only if $f(x) \geq f(c)$ for all x in some open interval.

A function f has a local maximum or local minimum at an endpoint c if the appropriate inequality holds for all x in some half-open domain interval containing x.

- Local extrema are also called **relative extrema**.
- An absolute extremum is also a local extremum , because being an extreme value overall makes it an extreme value in its immediate neighbourhood. The list of local extrema must include the absolute extreme value if a function has any.

Finding Extreme Values

- The interior domain points where a function has local extreme values are points where f' is zero of f' does not exist.

> Theorem: LOCAL EXTREME VALUES
>
> If a function f has a local maximum value OR a local minimum value at an interior point c of its domain, and if f' exists at c, then
>
> **f' (c) = 0**

> DEFINTION: **CRITICAL POINT**
>
> A point in the interior of the domain of a function f at which f' = 0 or f' does not exist is a **CRITICAL POINT** of f.

- Not every critical point or end point is necessarily an extreme value.
- Every extreme value occurs in a critical point

Example: Find the extreme values of the function and indicate where they occur.

$y = x^3 + x^2 - 8x + 9$