C12

Extreme Values of Functions

The Extreme Value Theorem:

If a function f is continuous on a closed interval [a,b], then f has both a maximum value and a minimum value on the interval.

Examples:

Definition: Local Extreme Values

Let c be an interior point of the domain of the function f. Then f(c) is a:

a) **Local maximum value** at c if and only if $f(x) \leq f(c)$ for all x in some open interval containing c.
b) **Local minimum value** at c if and only if $f(x) \geq f(c)$ for all x in some open interval containing c.

A function f has a local maximum or local minimum at an endpoint c if the appropriate inequality holds for x in some half-open domain interval containing c.
Note: Local extrema are also called relative extrema. An absolute extremum is also a local extremum, because being an extreme value for the entire function makes it also an extreme value in its immediate neighbourhood. Hence, a list of local extrema will automatically include absolute extrema if there are any.

Example:

Theorem: Local Extreme Values

If a function f has a local maximum value or a local minimum value at an interior point c of its domain, and if f' exists at c, then

$$f'(c) = 0$$

Definition: Critical Point

A point in the interior of the domain of a function f at which $f' = 0$ or f' does not exist is a critical point.

Note: any extremum (local or absolute) can only occur at a critical point or at end points. However, not every critical point or end point means that an extremum occurs there; for example, a critical point can be a point where a vertical tangent exist or a graph changes from opening down to opening up or vice versa.

Examples:

HW: p193-195 Answer at least 15 questions.