KEY

PRECALCULUS 12 EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Name________________________ Date______________ /47

Multiple Choice: Read each question carefully and circle the correct answer.

2 1. If $\log_2 3 = x$, then $\log_2 8\sqrt{3}$ can be represented as an algebraic expression, in terms of x, as

A. $\frac{1}{2}x + 8$

B. $\frac{1}{2}x + 3$

C. $2x + 8$

D. $2x + 3$

$\log_2 8\sqrt{3} = \log_2 8 + \frac{1}{2}\log_2 3$
$= 3\log_2 2 + \frac{1}{2}x$
$= 3 + \frac{1}{2}x$

1 2. The logarithm $\log_3 \frac{x^p}{x^q}$ is equal to

A. $(p-q)\log_3 x$

B. $\frac{p}{q}$

C. $p - q$

D. $\frac{p}{q}\log_3 x$

$\log_3 x^p - \log_3 x^q = p\log_3 x - q\log_3 x$
$= (p-q)(\log_3)$

1 3. Change to logarithmic form: $2^{-3} = \frac{1}{8}$

A. $\log_2 \frac{1}{8}$ = -3

B. $\log_{-3} 8$

C. $\log_8 \frac{2}{3}$

D. $\log 2 = \log \frac{1}{8}$

$\log_2 \frac{1}{8} = -3$

1 4. Change $\log_2(3x) = 5$ to exponential form

A. $3x = 2^5$

B. $3x = 5^2$

C. $2 = 3x^5$

D. $2 = (3x)^5$

$2^5 = 3x$

/5

$\frac{1}{2} - \log_{16}(x-3) = \log_{16} x$

$\frac{1}{2} = \log_{16}[(x-3)x]$

2 5. When solving: $\frac{1}{2} - \log_{16}(x-3) = \log_{16} x$, **the proposed** solution is

A. -1 and 4

B. 4

C. -4 and 1

D. -4

$0 = x^2 - 3x - 4$

$0 = (x-4)(x+1)$

$\frac{1}{2} = \log_{16}[x(x-3)]$

$16^{\frac{1}{2}} = x(x-3)$

$4 = x^2 - 3x$

- For an extra 2marks, identify whether the proposed is solution valid and explain why.

NPV: $x - 3 \le 0$, $x \le 3$

B2 $x \le 0$

$-1 \le 0$ ✗

$4 \ge 0$ ✓

$4 \ge 3$ ✓

Only 4 is valid

∴ -1 is extraneous because it is an NPV

1 6. Evaluate $\log_5 \sqrt{5^3}$

A. $\frac{1}{6}$

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 6

$\log_5 5^{\frac{3}{2}} = \frac{3}{2}\log_5 5$

$= \frac{3}{2}(1)$

$= \frac{3}{2}$

2 7. The pH of a solution is defines and pH$= -\log[H^+]$is the hydrogen ion concentration, in moles per litre. Acetic acid has a pH of 2.9. Formic acid is 4 times as concentrated as acetic acid. What is the pH of formic acid?

A. 1.1

B. 2.3

C. 3.5

D. 6.9

$pH_A = 2.9$

$pH_F = ?$

$4[H^+]_A = [H^+]_F$

$[H^+] = 10^{-pH}$

$4 = \frac{[H^+]_F}{[H^+]_A} = \frac{10^{-pH_F}}{10^{-pH_A}} \Rightarrow 4 = 10^{-pH_F + pH_A}$

$4 = 10^{-pH_F + 2.9}$

$4 = 10^{-pH_F} \cdot 10^{2.9}$

$10^{-pH_F} = \frac{4}{10^{2.9}}$

$\log\left(\frac{4}{10^{2.9}}\right) = -pH_F$

$pH_F = 2.3$

2 8. The exponential form of $k = -\log_h 5$ is

A. $h^k = \frac{1}{5}$

B. $k^h = \frac{1}{5}$

C. $h^k = -5$

D. $k^h = -5$

$h^{-k} = 5$

$k = \log_h 5^{-1}$

$k = \log_h \frac{1}{5}$

$h^k = \frac{1}{5}$

7

VS by $\frac{1}{2}$ HT left by 7

$y = \frac{1}{2} \log_3 (x+7)$

2 9. The effect on the graph of $\log_3 x$ if it is transformed to $y = \log_3 \sqrt{x+7}$ can be described as

A. a vertical stretch about the x-axis by a factor of $\frac{1}{2}$ and a vertical translation of 7 units up

B. a vertical stretch about the x-axis by a factor of $\frac{1}{2}$ and a horizontal translation of 7 units left

C. a horizontal stretch about the y-axis by a factor of $\frac{1}{2}$ and a vertical translation of 7 units up

D. a horizontal stretch about the y-axis by a factor of $\frac{1}{2}$ and a horizontal translation of 7 units left

1 10. Evaluate $\log_3 81$

A. 0

B. 3

C. 4

D. $\sqrt[3]{81}$

$\log_3 81 = x$

$3^x = 8$

$3^4 = 8 \rightarrow x = 4$

1 11. Solve the equation $-2\log_5 7x = 2$

A. 35

B. $\frac{7}{5}$

C. $\frac{5}{7}$

D. $\frac{1}{35}$

$\frac{-2 \log_5 7x}{-2} = \frac{2}{-2}$

$\log_5 7x = -1$

$5^{-1} = 7x$

$\frac{1}{5} = 7x$

$1 = 35x$

$x = \frac{1}{35}$

1 12. Solve for x: 2log x = log 32 + log 2

A. 64

B. 8

C. 16

D. 32

$2 \log x = 5 \log 2 + \log 2$

$\frac{2 \log x}{2} = \frac{6 \log 2}{2}$

$\log x = 3 \log 2$

$\log x = \log 8$

$x = 2^3$

/5

1 13. Determine the base of the following logarithm: $\log_2 5$

A. 2

B. 32

C. 5

D. 25

$x = 2^{y-3} + 1$

$x - 1 = 2^{(y-3)}$

$\log_2 (x-1) = y - 3$

$y = \log_2 (x-1) + 3$

2 14. Find the inverse of $y = 2^{x-3} + 1$

A. $y = \frac{1}{2^{x-3}+1}$

B. $y = \log_2(x-1) + 3$

C. $y = \log_2(x+1) - 3$

D. $y = \log_2(x-3) + 1$

2 15. Solve for x if $16^{x-7} + 5 = 24$

A. 10.21

B. 8.06

C. -5.82

D. no real solutions

$16^{x-7} + 5 = 24$

$16^{x-7} = 19$

$\log_{16} 19 = x - 7$

let $A = (x-7)$

$\log_{16} 19 = A$

$A = \frac{\log 19}{\log 16}$

$A = 1.061981874$

$x - 7 = 1.06198\ldots$

$\rightarrow x = 8.06198\ldots$

/5

Short Answer

Please SHOW ALL WORK for full marks.

1. a) Rewrite the function $y = 2^{-2x+4} + 6$ in the form $f(x) = a(2)^{b(x-h)} + k$: $f(x) = 1 \cdot 2^{-2(x-2)} + 6$ 1

b) Describe the transformations that must be applied to the graph of $y = 2^x$ to obtain the graph of the given function. **Three points have to be exact.** You may find mapping notation useful.

- R in y-axis
- HSC by a factor of $\frac{1}{2}$
- HT right by 2 units
- VT up by 6 units

$(x,y) \to (-\frac{1}{2}x + 2, y + 6)$

$(0,1) \to (2, 7)$

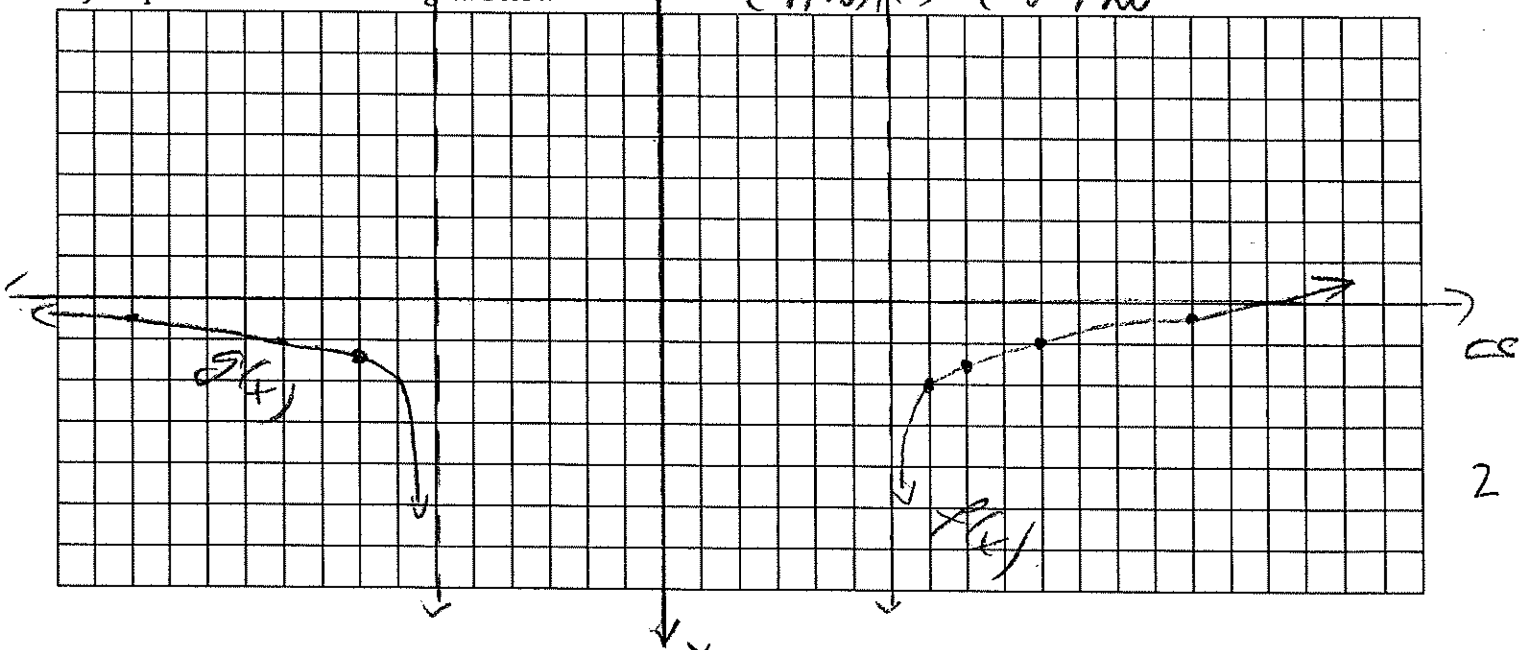
$(1,2) \to (1.5, 8)$ 3

$(2,4) \to (1, 10)$

$(3,8) \to (0.5, 14)$

$(4,16) \to (0, 20)$

c) Graph the function on the grid below



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d) Determine the equation of the function g(x) that results after the graph in part c) is reflected in the x-axis.

$g(x) = -2^{-2(x-2)} - 6$ 1

e) Graph the function from d) on the same grid and label the graph g(x)

2

/9

$(x,y) \rightarrow (\frac{1}{2}x+3, -y-1)$

$(1,0) \rightarrow (3.5, -1)$

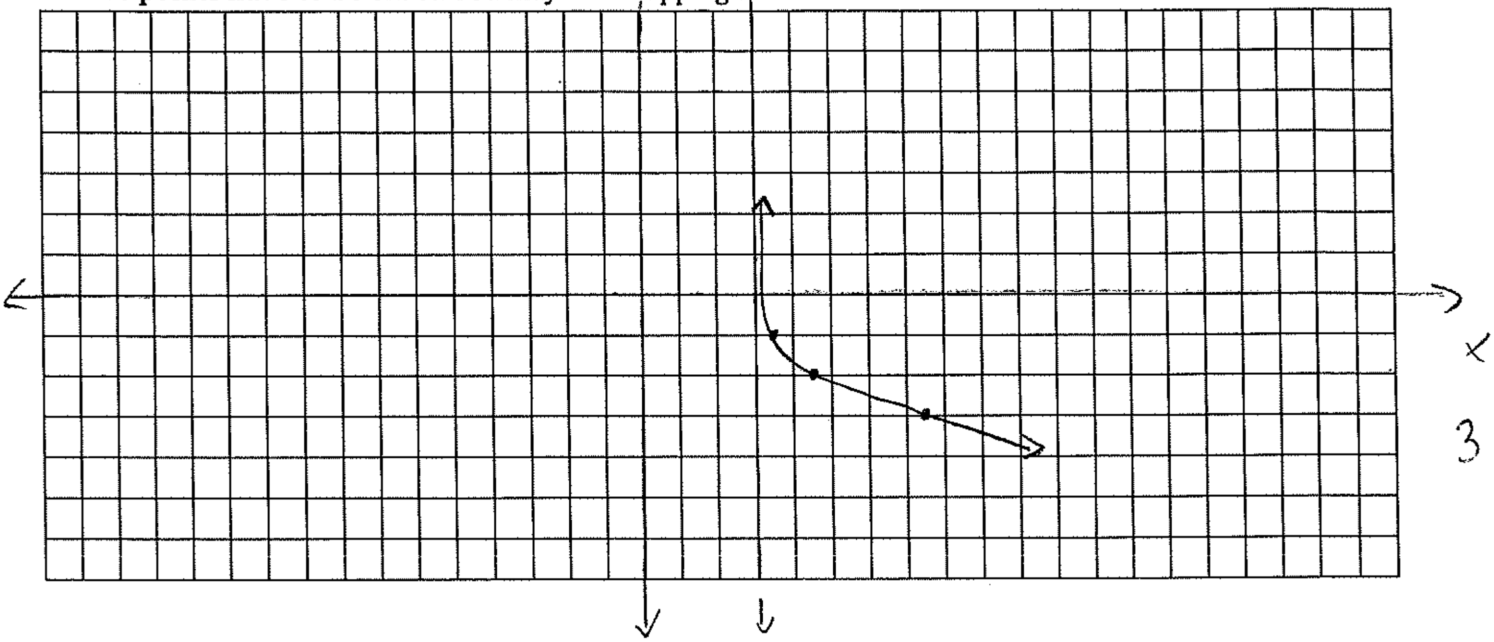
$(3,1) \rightarrow (4.5, -2)$

$(9,2) \rightarrow (7.5, -3)$

VA : $x = 3$

2. a) Sketch a graph of a function $f(x) = -\log_3(2x-6) - 1$ $= -\log_3(2(x-3)) - 1$ i

Three points have to be exact. You may find mapping notation useful.



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Identify the domain $\{x \mid x>3, x \in \mathbb{R}\}$ range $\{y \mid y \in \mathbb{R}\}$ of f(x) 3

and the equation of any asymptotes if they exist VA: $x = 3$

3. Write as a single logarithm: $4\log a^2 - 2\log a$

$= \log a^8 - \log a^2$

$= \log \frac{a^8}{a^2}$

$= \underline{\log a^6 = 6\log a}$ 2

4. Solve: $-10 + \log_3(x+3) = -10$

$\log_3(x+3) = 0$

$3^0 = x+3$

$1 = x+3$ 2

$\boxed{-2 = x}$

5. The sound intensity level β, in decibels is defined as $\beta = 10\log\frac{I}{I_0}$, where I is the intensity of the sound, in watts per square meter, and I_0, the threshold of hearing, is $10^{-12}W/m^2$. A refrigerator in the kitchen of a restaurant has a decibel level of 45dB. How many times as intense is the sound of the refrigerator as the sound of a chainsaw at 85dB?

$$\beta = 10\log\frac{I}{I_0}$$

$\beta_R = 45dB$

$\beta_C = 85dB$

Find $\frac{I_R}{I_C}$

$$\frac{\beta}{10} = \log I - \log I_0$$

$$\log I = \frac{\beta}{10} + \log I_0$$

$$I = 10^{\left(\frac{\beta}{10} + \log I_0\right)}$$

$$\frac{I_R}{I_C} = \frac{10^{\left(\frac{45}{10} + \log I_0\right)}}{10^{\left(\frac{85}{10} + \log I_0\right)}}$$

$$= 10^{4.5-8.5}$$

$$= 10^{-4}$$

$$= 0.0001$$

∴ The sound of the refrigerator is 0.0001 times as intense as the sound of the chain saw. In other words, the chainsaw is 10 000 as intense as the refrigerator

/5

NPV: $x = 1$
$x \leq 0$
$x + 6 \leq 0$
$x \leq -6$

$\rightarrow$ Restrictions: $x \neq 1$.
$x > 0$.
$x > -6$

BONUS: Only <u>completely correct</u> solutions will earn the bonus marks. You may use an additional sheet of paper.

1. Solve $\log_2(\log_x(x+6)) = 1$ Remember to clearly identify restrictions and valid solutions

B3

$2^1 = \log_x(x+6)$

$x^2 = x + 6$

$0 = x^2 - x - 6$

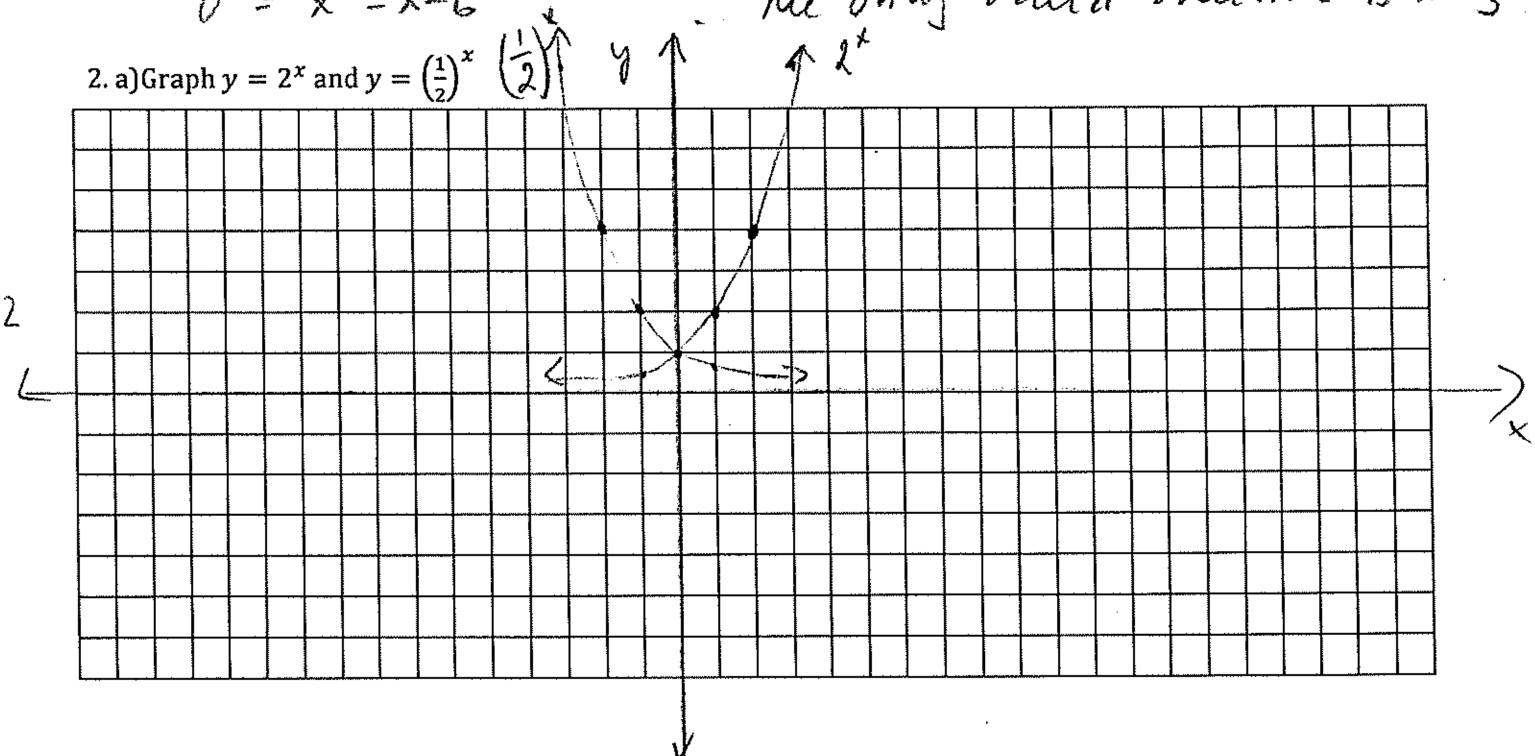
$\rightarrow 0 = (x-3)(x+2)$

$x = 3$ $\quad$ $x = -2$ ↑ extraneous

$\therefore$ the only valid solution is $x = 3$

2. a) Graph $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$

B2



b) Describe the relationship between the two functions **<u>in two different ways</u>**

B2

- Reflected in the y-axis
- $2^x = \left(\frac{2}{1}\right)^x \rightarrow$ reciprocate $\quad \frac{1}{\left(\frac{2}{1}\right)^x} = \frac{1}{2^x} = \left(\frac{1}{2}\right)^x$

3. If y = log x, then calculate y + 2 and express it as a single logarithm

$y + 2 = \log x + 2$

$= \log x + 2\log 10$

B1

$= \log x + \log 100$

$= \log 100x$