

Key

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Function	Property	Symmetry
Even	$f(x) = f(-x)$ Reflection in the y-axis looks exactly like the original	- About the y-axis
Examples:	$y = x^2$	$f(x) = \cos x$
	$y = x $	$f(x) = \sec x$
	$f(x) = x^4 + 5$	$f(x) = \frac{1}{x^2}$
Odd	$f(-x) = -f(x)$ Reflection in the x-axis looks the same as reflection in the y-axis	- About the origin
Examples	$f(x) = \frac{1}{x}$	$f(x) = x$
	$f(x) = \sin x$	$f(x) = x^3$
	$f(x) = \csc x$	$f(x) = -x^5$
Not even nor odd	$f(x) \neq f(-x)$ $f(-x) \neq -f(x)$	none
Examples	$y = \frac{1}{2}x + 4$	$f(x) = x - 2 $
	$f(x) = (x - 2)^2 + 4$	$f(x) = \frac{1}{(x + 1)^2}$
	$f(x) = x^3 + 1$	$f(x) = \sqrt{x + 1}$

You may find it helpful, when answering this "even or odd" type of question, to write down $f(x)$ and $-f(x)$ explicitly, and then compare them to whatever you get for $f(-x)$. This can help you make a confident determination of the correct answer.

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1. A) Using algebraic approach and properties of odd and even functions, determine whether the given function is even, odd or neither.
 B) State whether the function is symmetric. Specify about what axis or point it is symmetric.
 Check your answers by graphing each function.

a) $f(x) = x^6 - x^4 + x^2 + 1$

$$f(-x) = (-x)^6 - (-x)^4 + (-x)^2 + 1 = x^6 - x^4 + x^2 + 1 = f(x)$$

EVEN

b) $f(x) = 3x^8 - 2x^4$

$$f(-x) = 3(-x)^8 - 2(-x)^4 = 3x^8 - 2x^4 = f(x)$$

EVEN

c) $f(x) = \frac{2}{x^2}$

$$f(-x) = \frac{2}{(-x)^2} = \frac{2}{x^2} = f(x)$$

EVEN

d) $f(x) = x^3 + x^2$

$$f(-x) = (-x)^3 + (-x)^2 = -x^3 + x^2 \neq f(x)$$

$$-f(x) = -(x^3 + x^2) = -x^3 - x^2 \neq f(-x)$$

NEITHER

e) $f(x) = \frac{x^2}{1-x^2}$

$$f(-x) = \frac{(-x)^2}{1-(-x)^2} = \frac{x^2}{1-x^2} = f(x)$$

EVEN

f) $f(x) = |x|$

$$f(-x) = |-x| = |x| = f(x)$$

EVEN

g) $f(x) = x|x|$

$$f(-x) = -x|-x| = -x|x| \neq f(x)$$

$$-f(x) = -(x|x|) = -x|x| = f(-x)$$

ODD