C12

Algebraic Approach to Limits

1. $\lim_{x \to 5} 12 = \boxed{12}$

2. $\lim_{x \to 0} 2\pi = \boxed{2\pi}$

3. $\lim_{x \to 2} 4x = 4(2) = \boxed{8}$

4. $\lim_{x \to 5} 3x^2 - 4x - 1 = 3(5)^2 - 4(5) - 1 = \boxed{54}$

#5 $\lim_{x \to 0} 5x^3 - 7x^3 + 2^x - 2 = 5(0)^3 - 7(0)^3 + 2^0 - 2 = 0 - 0 + 1 - 2 = \boxed{-1}$

#6 $\lim_{y \to -1} 3y^4 - 6y^3 - 2y = 3(-1)^4 - 6(-1)^3 - 2(-1) = 3 + 6 + 2 = \boxed{11}$

#7 $\lim_{x \to 4} \dfrac{2x-4}{x-1} = \dfrac{2(4)-4}{4-1} = \boxed{\dfrac{4}{3}}$

#8 $\lim_{x \to -2} \dfrac{x^2+4x+4}{x^2} = \dfrac{(-2)^2+4(-2)+4}{(-2)^2} = \dfrac{0}{4} = \boxed{0}$

#9 $\lim_{x \to 1} \dfrac{2x-2}{x-1} = \lim_{x \to 1} \dfrac{2\cancel{(x-1)}}{\cancel{(x-1)}} = \boxed{2}$

#10 $\lim\limits_{x\to 4} \frac{x^2-16}{x-4} = \lim\limits_{x\to 4} \frac{\cancel{(x-4)}(x+4)}{\cancel{(x-4)}} = \boxed{8}$

#11 $\lim\limits_{x\to -2} \frac{t^3+8}{t+2} = \lim\limits_{x\to -2} \frac{\cancel{(t+2)}(t^2-2t+4)}{\cancel{(t+2)}} = (-2)^2-2(-2)+4 = \boxed{12}$

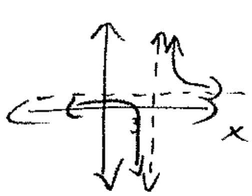
#12 $\lim\limits_{x\to 2} \frac{x^2-4x+4}{x^2+x-6} = \lim\limits_{x\to 2} \frac{(x-2)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}} = \lim\limits_{x\to 2} \frac{x-2}{x+3} = \frac{2-2}{2+3} = \frac{0}{5} = \boxed{0}$

#13 $\lim\limits_{x\to -1} \frac{x^2+6x+5}{x^2-3x-4} = \lim\limits_{x\to -1} \frac{\cancel{(x+1)}(x+5)}{(x-4)\cancel{(x+1)}} = \lim\limits_{x\to -1} \frac{x+5}{x-4} = \frac{-1+5}{-1-4} = \frac{4}{-5} = \boxed{-4/5}$

#14 $\lim\limits_{x\to 2} \frac{x^3+x^2-4x-4}{x^4-16} = \lim\limits_{x\to 2} \frac{x^2(x+1)-4(x+1)}{(x^2-4)(x^2+4)} = \lim\limits_{x\to 2} \frac{\cancel{(x^2-4)}(x+1)}{\cancel{(x^2-4)}(x^2+4)} =$

$= \lim\limits_{x\to 2} \frac{x+1}{x^2+4} = \frac{2+1}{2^2+4} = \boxed{\frac{3}{8}}$

#15 $\lim\limits_{x\to 3} \frac{x}{(x-3)} = \lim\limits_{x\to 3} \frac{x-3+3}{x-3} = \lim\limits_{x\to 3} \frac{x-3}{x-3} + \frac{3}{x-3} = \lim\limits_{x\to 3} \frac{3}{x-3} + 1$



$\begin{cases} \lim\limits_{x\to 3^-} \frac{x}{x-3} = -\infty \\ \lim\limits_{x\to 3^+} \frac{x}{x-3} = +\infty \end{cases}$

$-\infty \neq \infty \quad \therefore \lim\limits_{x\to 3} \frac{x}{x-3} = \boxed{DNE}$

$\Rightarrow$ VA at $x = NPV \rightarrow$ find both one-sided limits

#16 $\lim_{x \to 5} \frac{x}{x^2-25} = \lim_{x \to 5} \frac{x}{(x-5)(x+5)}$

VA: $x =$ NPV

$$\lim_{x \to 5^+} \frac{x}{x^2-25} = \frac{\sim 5.0\ldots1}{\sim \text{very small } (+)\#} = +\infty$$

$+\infty \neq -\infty$

$$\lim_{x \to 5^-} \frac{x}{x^2-25} = \frac{\sim 4.9\ldots9}{\sim \text{very small } (-)\#} = -\infty$$

$\therefore \lim_{x \to 5} \frac{x}{x^2-25} = \boxed{DNE}$

#17 $\lim_{y \to 6} \frac{y+6}{y^2-36} = \lim_{y \to 6} \frac{\cancel{(y+6)}}{\cancel{(y+6)}(y-6)} = \lim_{y \to 6} \frac{1}{y-6} = \left[\frac{1}{0}\right]$

VA: $y = 6$

$$\lim_{y \to 6^+} \frac{1}{y-6} = \frac{1}{\text{very small}(+)\#} = +\infty$$

$+\infty \neq \infty$

$$\lim_{y \to 6^-} \frac{1}{y-6} = \frac{1}{\text{very small } (-)\#} = -\infty$$

$\therefore \lim_{y \to 6} \frac{y+6}{y^2-36} = \boxed{DNE}$

#18 $\lim_{x\to 4} \frac{3-x}{x^2-2x-8} = \lim_{x\to 4} \frac{(3-x)}{(x-4)(x+2)}$

$\Rightarrow$ VA at $x=4$ and $x=-2$

$$\begin{cases} \lim_{x\to 4^+} \frac{3-x}{(x-4)(x+2)} = \frac{\sim -1}{\text{very small}(+)\ \#} = -\infty \\ \lim_{x\to 4^-} \frac{3-x}{(x-4)(x+2)} = \frac{\sim -1}{\text{very small}(-)\ \#} = +\infty \end{cases}$$

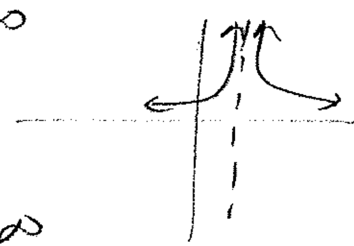
$-\infty \neq +\infty$

$\therefore \lim_{x\to 4} \frac{3-x}{x^2-2x-8} = \boxed{DNE}$

#19 $\lim_{x\to 1} \frac{4}{x^2-2x+1} = \lim_{x\to 1} \frac{4}{(x-1)^2}$

$\Rightarrow$ VA : $x=1$

$$\begin{cases} \lim_{x\to 1^+} \frac{4}{(x-1)^2} = \frac{4}{\text{very small}(+)\ \#} = +\infty \\ \lim_{x\to 1^-} \frac{4}{(x-1)^2} = \frac{4}{\text{very small}(+)\ \#} = +\infty \end{cases}$$



$\therefore \lim_{x\to 1} \frac{4}{x^2-2x+1} = \boxed{+\infty}$

#20 $\lim_{x \to 5} \frac{x}{|x-5|}$

VA: $x = 5$

$\lim_{x \to 5^+} \frac{x}{|x-5|} = \frac{\sim 5}{\text{very small}(+)\#} = +\infty$

$+\infty = +\infty$

$\lim_{x \to 5^-} \frac{x}{|x-5|} = \frac{\sim 5}{\text{very small}(+)\#} = +\infty$

$\therefore \lim_{x \to 5} \frac{x}{|x-5|} = \boxed{\infty}$

#21 $\lim_{x \to 3} \frac{-x^2}{(x-3)^2}$

VA: $x = 3$

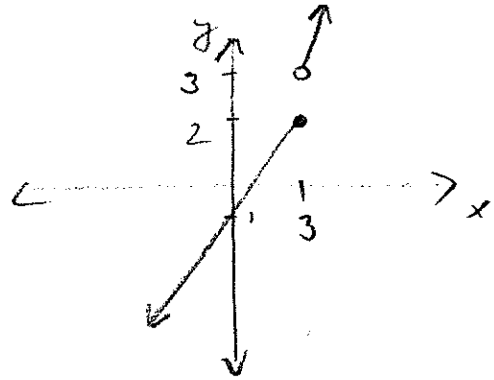
$\lim_{x \to 3^+} \frac{-x^2}{(x-3)^2} = \frac{\sim -9}{\text{very small}(+)\#} = -\infty$

$-\infty = -\infty$

$\lim_{x \to 3^-} \frac{-x^2}{(x-3)^2} = \frac{\sim -9}{\text{very small}(+)\#} = -\infty$

$\therefore \lim_{x \to 3} \frac{-x^2}{(x-3)^2} = \boxed{-\infty}$

#22 $f(x) = \begin{cases} x-1, & x \le 3 \\ 2x-3, & x > 3 \end{cases}$



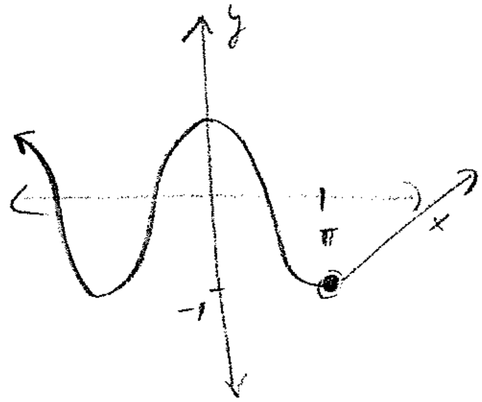
$\lim_{x \to 3} f(x) = \boxed{DNE}$ because

$\lim_{x \to 3^+} f(x) = 2(3)-3 = 3$

$3 \neq 2$

$\lim_{x \to 3^-} f(x) = 3-1 = 2$

#23 $f(x) = \begin{cases} \cos x - \sin \pi \\ = \cos x - 0 \\ = \cos x & ; \ x \le \pi \\ x - \pi - 1 & ; \ x > \pi \\ = x - 4.14 \end{cases}$



$\lim_{x \to \pi^+} f(x) = \pi - \pi - 1 = -1$

$\lim_{x \to \pi^+} f(x) = \cos \pi = -1$

$-1 = -1$ $\therefore \lim_{x \to \pi} f(x) = \boxed{-1}$

#24 $f(x) = \begin{cases} x^3 + x, & x \le -1 \\ -2^{-x}, & x > -1 \end{cases}$

$\lim_{x \to -1^+} f(x) = -2^{-(-1)} = -2^{+1} = -2$

$-2 = -2$

$\lim_{x \to -1^-} f(x) = (-1)^3 + (-1) = -1-1 = -2$

$\therefore \lim_{x \to -1} f(x) = \boxed{-2}$

#25 $f(x)=\begin{cases} \dfrac{x-2}{x-1}, & x<1 \\ \dfrac{x}{x-1}, & >1 \end{cases}$

$\lim_{x\to 1^+} f(x) = \dfrac{\sim +1}{\text{very small}(+)\#} = +\infty$

$+\infty = +\infty$

$\lim_{x\to 1^-} f(x) = \dfrac{\sim 1}{\text{very small}(+)\#} = +\infty$

$\boxed{VA: x=1}$

$\lim_{x\to 1} f(x) = \boxed{+\infty}$

#26 $\lim_{x\to 0} \dfrac{\sqrt{x+4}-2}{x} = \lim_{x\to 0} \dfrac{(\sqrt{x+4}-2)(\sqrt{x+4}+2)}{x(\sqrt{x+4}+2)} = \lim_{x\to 0} \dfrac{x+4-4}{x(\sqrt{x+4}+2)} =$

$= \lim_{x\to 0} \dfrac{\cancel{x}}{\cancel{x}\sqrt{x+4}+2} = \dfrac{1}{\sqrt{4}+2} = \boxed{\dfrac{1}{4}}$

#27 $\lim_{x\to 4} \dfrac{x^2-2x-8}{[\cos \pi x](x-4)} = \lim_{x\to 4} \dfrac{\cancel{(x-4)}(x+2)}{(\cos \pi x)\cancel{(x-4)}} = \dfrac{6}{1} = \boxed{6}$

#28 $f(x)=\begin{cases} x^2-2x-3, & x\neq 2 \\ k-3, & x=2 \end{cases}$

$\lim_{x\to 2} f(x) = 2^2-2(2)-3 = -3$

$\Rightarrow f(2) = -3$

$\lim_{x\to 2} f(x) = f(2)$

$k-3=-3$

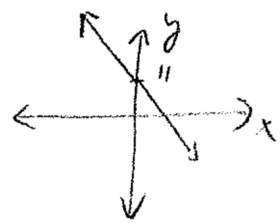
$\boxed{k=0}$

#29 $f(x) = \begin{cases} \dfrac{x^2-49}{x-7}, & x \neq 7 \\ k^2-2, & x = 7 \end{cases}$ $\lim\limits_{x \to 7} \dfrac{x^2-49}{x-7} = \lim\limits_{x \to 7} \dfrac{\cancel{(x-7)}(x+7)}{\cancel{(x-7)}} = 14$

$\lim\limits_{x \to 7} f(x) = f(7)$

$k^2 - 2 = 14$
$k^2 = 16$
$\boxed{k = \pm 4}$

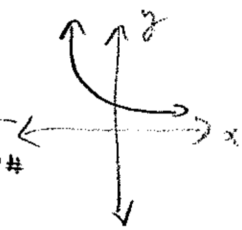
#30 $\lim\limits_{x \to \infty} 6 = \boxed{6}$



#31 $\lim\limits_{x \to -\infty} (11 - 2x) = \boxed{+\infty}$ (decreasing)

#32 $\lim\limits_{x \to \infty} (0.2x^4 - x^2 - 9) = \boxed{+\infty}$ (opens up)

#33 $\lim\limits_{x \to \infty} 2^{-x} = \lim\limits_{x \to \infty} \dfrac{1}{2^x} = \dfrac{1}{\text{very large}(+)\#}$



$= \boxed{0}$

#34 $\lim\limits_{x \to \infty} \dfrac{2x-3}{4x+5} = \lim\limits_{x \to \infty} \dfrac{2x}{4x} = \boxed{\dfrac{1}{2}}$

#35 $\lim_{x\to-\infty} \frac{7-3x^3}{2x^3+1} = \lim_{x\to-\infty} \frac{-3\cancel{x^3}}{2\cancel{x^3}} = \boxed{-1.5}$

#36 $\lim_{x\to\infty} \frac{2}{5x-4} = \lim_{x\to\infty} \frac{2}{5x} = \frac{2}{\text{very large(+)\#}} = \boxed{0}$

#37 $\lim_{x\to\infty} \frac{4x5}{1-5x^3} = \lim_{x\to\infty} \frac{4x\cancel{5}^{2}}{-5\cancel{x^3}} = \lim_{x\to\infty} \frac{4x^2}{-5} = \frac{\text{very large(+) \#}}{-5}$

$= \boxed{-\infty}$

#38 $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+4}} = \left[\frac{\infty}{\infty}\right]$ indeterminate form

end behavior model: $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+4}} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2}} = \lim_{x\to\infty} \frac{x}{|x|} = \boxed{1}$

#39 $\lim_{x\to-\infty} \frac{x}{\sqrt{x^2+4}} = \lim_{x\to-\infty} \frac{x}{\sqrt{x^2}} = \lim_{x\to-\infty} \frac{x}{|x|} = \boxed{-1}$

#40 $$\lim_{x\to-\infty} \frac{\sqrt{3x^2+x}}{x^2-1} = \lim_{x\to-\infty} \frac{\sqrt{3x^2}}{x^2}$$

$$= \lim_{x\to-\infty} \frac{\sqrt{3}\sqrt{x^2}}{x^2} = \lim_{x\to-\infty} \frac{\sqrt{3}|x|}{x^2} = \lim_{x\to-\infty} \frac{\sqrt{3}}{x} = \boxed{0}$$

#41 $$\lim_{x\to-\infty} \frac{\sqrt{3x^2+x}}{x-1} = \lim_{x\to-\infty} \frac{\sqrt{3}\,|x|}{x} = \sqrt{3}\,(-1) = \boxed{-\sqrt{3}}$$