

Calculus 1 - Limits  
Worksheet 4  
Evaluating Limits by Factoring,  
Part 2

Calculus 1 - Limits - Worksheet 4 – Evaluating Limits by Factoring, Part 2

1. Evaluate this limit.

$$\lim_{x \rightarrow 0} \frac{(4 + x)^2 - 16}{x}$$

2. Evaluate this limit.

$$\lim_{x \rightarrow 0} \frac{5x^3 + 8x^2}{3x^4 - 16x^2}$$

3. Evaluate this limit.

$$\lim_{x \rightarrow 0} \frac{(2 + x)^3 - 8}{x}$$

4. Evaluate this limit.

$$\lim_{x \rightarrow 1} \frac{4x^3 - 2x^2 + x - 3}{x - 1}$$

5. Evaluate this limit.

$$\lim_{x \rightarrow 4} \frac{2x^2 - 12x + 16}{x^2 - x - 12}$$

6. Evaluate this limit.

$$\lim_{x \rightarrow -3} \frac{x^2 + 2x - 3}{-3x - 9}$$

7. Evaluate this limit.

$$\lim_{x \rightarrow -2} \frac{-x^2 + x + 6}{x^2 - 2x - 8}$$

8. Evaluate this limit.

$$\lim_{x \rightarrow 16} \frac{16 - x}{4 - \sqrt{x}}$$

9. Evaluate this limit.

$$\lim_{x \rightarrow -4} \frac{8x + 32}{x^2 + 5x + 4}$$

10. Evaluate this limit.

$$\lim_{x \rightarrow -3} \frac{9x + 27}{x^2 + 5x + 6}$$

11. Evaluate this limit.

$$\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{-4x^2 - 16x - 12}$$

12. Evaluate this limit.

$$\lim_{x \rightarrow 3} \frac{x^3 - 9x}{3x^2 - 6x - 9}$$

13. Evaluate this limit.

$$\lim_{x \rightarrow 25} \frac{5 - \sqrt{x}}{25 - x}$$

14. Evaluate this limit.

$$\lim_{x \rightarrow -2} \frac{2x^2 + 10x + 12}{x^2 + 3x + 2}$$

15. Evaluate this limit.

$$\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x^2 + 8x + 15}$$

16. Evaluate this limit.

$$\lim_{x \rightarrow -4} \frac{x^2 + 5x + 4}{x^2 + 9x + 20}$$

17. Evaluate this limit.

$$\lim_{x \rightarrow 5} \frac{2x^2 - 4x - 30}{3x^2 - 18x + 15}$$

18. Evaluate this limit.

$$\lim_{x \rightarrow 0} \frac{16x^3 + 80x^2}{40x^3 - 80x^2}$$

19. Evaluate this limit.

$$\lim_{x \rightarrow -4} \frac{x^3 + 64}{x + 4}$$

20. Evaluate this limit.

$$\lim_{x \rightarrow -1} \frac{x^3 + x^2 - 9x - 9}{x^3 + x^2 - 4x - 4}$$



Answers - Calculus 1 - Limits - Worksheet 4 – Evaluating Limits by Factoring, Part 2

1. Evaluate this limit.

$$\lim_{x \rightarrow 0} \frac{(4 + x)^2 - 16}{x}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute 0 into the limit for  $x$ .

$$\lim_{x \rightarrow 0} \frac{(4 + x)^2 - 16}{x} = \frac{(4 + 0)^2 - 16}{0} = \frac{16 - 16}{0} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, multiply the numerator and simplify the limit.

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{(4 + x)^2 - 16}{x} &= \lim_{x \rightarrow 0} \frac{16 + 8x + x^2 - 16}{x} \\ &= \lim_{x \rightarrow 0} \frac{8x + x^2}{x} = \lim_{x \rightarrow 0} (8 + x) \end{aligned}$$

Substitute 0 into the limit for  $x$ .

$$\lim_{x \rightarrow 0} (8 + x) = 8 + 0 = 8$$

**Answer:**  $\lim_{x \rightarrow 0} \frac{(4+x)^2-16}{x} = 8$

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2. Evaluate this limit.

$$\lim_{x \rightarrow 0} \frac{5x^3 + 8x^2}{3x^4 - 16x^2}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute 0 into the limit for  $x$ .

$$\lim_{x \rightarrow 0} \frac{5x^3 + 8x^2}{3x^4 - 16x^2} = \frac{5(0)^3 + 8(0)^2}{3(0)^4 - 16(0)^2} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow 0} \frac{5x^3 + 8x^2}{3x^4 - 16x^2} = \lim_{x \rightarrow 0} \frac{x^2(5x + 8)}{x^2(3x^2 - 16)} = \lim_{x \rightarrow 0} \frac{5x + 8}{3x^2 - 16}$$

Substitute 0 into the limit for  $x$ .

$$\lim_{x \rightarrow 0} \frac{5x + 8}{3x^2 - 16} = \frac{5(0) + 8}{(3(0)^2 - 16)} = \frac{8}{-16} = -\frac{1}{2}$$

**Answer:**  $\lim_{x \rightarrow 0} \frac{5x^3 + 8x^2}{3x^4 - 16x^2} = -\frac{1}{2}$

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3. Evaluate this limit.

$$\lim_{x \rightarrow 0} \frac{(2+x)^3 - 8}{x}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute 0 into the limit for  $x$ .

$$\lim_{x \rightarrow 0} \frac{(2+x)^3 - 8}{x} = \frac{(2+0)^3 - 8}{0} = \frac{8 - 8}{0} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, multiply the numerator and simplify the limit.

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{(2+x)^3 - 8}{x} &= \lim_{x \rightarrow 0} \frac{x^3 + 6x^2 + 12x + 8 - 8}{x} \\ \lim_{x \rightarrow 0} \frac{x^3 + 6x^2 + 12x}{x} &= \lim_{x \rightarrow 0} (x^2 + 6x + 12) \end{aligned}$$

Substitute 0 into the limit for  $x$ .

$$\lim_{x \rightarrow 0} (x^2 + 6x + 12) = 0^2 + 6(0) + 12 = 12$$

**Answer:**  $\lim_{x \rightarrow 0} \frac{(2+x)^3 - 8}{x} = 12$

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4. Evaluate this limit.

$$\lim_{x \rightarrow 1} \frac{4x^3 - 2x^2 + x - 3}{x - 1}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute 1 into the limit for  $x$ .

$$\lim_{x \rightarrow 1} \frac{4x^3 - 2x^2 + x - 3}{x - 1} = \frac{4(1)^3 - 2(1)^2 + 1 - 3}{1 - 1} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{4x^3 - 2x^2 + x - 3}{x - 1} &= \lim_{x \rightarrow 1} \frac{(4x^2 + 2x + 3)(x - 1)}{x - 1} \\ &= \lim_{x \rightarrow 1} (4x^2 + 2x + 3) \end{aligned}$$

Substitute 1 into the limit for  $x$ .

$$= \lim_{x \rightarrow 1} (4x^2 + 2x + 3) = 4(1)^2 + 2(1) + 3 = 9$$

**Answer:**  $\lim_{x \rightarrow 1} \frac{4x^3 - 2x^2 + x - 3}{x - 1} = 9$

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5. Evaluate this limit.

$$\lim_{x \rightarrow 4} \frac{2x^2 - 12x + 16}{x^2 - x - 12}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute 4 into the limit for  $x$ .

$$\lim_{x \rightarrow 4} \frac{2x^2 - 12x + 16}{x^2 - x - 12} = \frac{2(4)^2 - 12(4) + 16}{4^2 - 4 - 12} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\begin{aligned} \lim_{x \rightarrow 4} \frac{2x^2 - 12x + 16}{x^2 - x - 12} &= \lim_{x \rightarrow 4} \frac{2(x^2 - 6x + 8)}{x^2 - x - 12} \\ &= \lim_{x \rightarrow 4} \frac{2(x - 2)(x - 4)}{(x - 4)(x + 3)} = \lim_{x \rightarrow 4} \frac{2(x - 2)}{x + 3} \end{aligned}$$

Substitute 4 into the limit for  $x$ .

$$= \lim_{x \rightarrow 4} \frac{2(x - 2)}{x + 3} = \frac{2(4 - 2)}{4 + 3} = \frac{4}{7}$$

**Answer:**  $\lim_{x \rightarrow 4} \frac{2x^2 - 12x + 16}{x^2 - x - 12} = \frac{4}{7}$

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6. Evaluate this limit.

$$\lim_{x \rightarrow -3} \frac{x^2 + 2x - 3}{-3x - 9}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute  $-3$  into the limit for  $x$ .

$$\lim_{x \rightarrow -3} \frac{x^2 + 2x - 3}{-3x - 9} = \frac{(-3)^2 + 2(-3) - 3}{-3(-3) - 9} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow -3} \frac{x^2 + 2x - 3}{-3x - 9} = \lim_{x \rightarrow -3} \frac{(x + 3)(x - 1)}{-3(x + 3)} = \lim_{x \rightarrow -3} \frac{x - 1}{-3}$$

Substitute  $-3$  into the limit for  $x$ .

$$\lim_{x \rightarrow -3} \frac{x - 1}{-3} = \frac{-3 - 1}{-3} = \frac{-4}{-3} = \frac{4}{3}$$

**Answer:**  $\lim_{x \rightarrow -3} \frac{x^2 + 2x - 3}{-3x - 9} = \frac{4}{3}$

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7. Evaluate this limit.

$$\lim_{x \rightarrow -2} \frac{-x^2 + x + 6}{x^2 - 2x - 8}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute  $-2$  into the limit for  $x$ .

$$\lim_{x \rightarrow -2} \frac{-x^2 + x + 6}{x^2 - 2x - 8} = \frac{-(-2)^2 + (-2) + 6}{(-2)^2 - 2(-2) - 8} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\begin{aligned} \lim_{x \rightarrow -2} \frac{-x^2 + x + 6}{x^2 - 2x - 8} &= \lim_{x \rightarrow -2} \frac{-(x^2 - x - 6)}{x^2 - 2x - 8} \\ &= \lim_{x \rightarrow -2} \frac{-(x - 3)(x + 2)}{(x - 4)(x + 2)} = \lim_{x \rightarrow -2} \frac{-(x - 3)}{x - 4} \end{aligned}$$

Substitute  $-2$  into the limit for  $x$ .

$$\lim_{x \rightarrow -2} \frac{-(x - 3)}{x - 4} = \frac{-(-2 - 3)}{-2 - 4} = \frac{5}{-6} = -\frac{5}{6}$$

**Answer:**  $\lim_{x \rightarrow -2} \frac{-x^2 + x + 6}{x^2 - 2x - 8} = -\frac{5}{6}$

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8. Evaluate this limit.

$$\lim_{x \rightarrow 16} \frac{16 - x}{4 - \sqrt{x}}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute 16 into the limit for  $x$ .

$$\lim_{x \rightarrow 16} \frac{16 - x}{4 - \sqrt{x}} = \frac{16 - 16}{4 - \sqrt{16}} = \frac{16 - 16}{4 - 4} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, rationalize the denominator and simplify the limit.

$$\begin{aligned} \lim_{x \rightarrow 16} \frac{16 - x}{4 - \sqrt{x}} &= \lim_{x \rightarrow 16} \frac{(16 - x)}{(4 - \sqrt{x})} \cdot \frac{(4 + \sqrt{x})}{(4 + \sqrt{x})} \\ &= \lim_{x \rightarrow 16} \frac{(16 - x)(4 + \sqrt{x})}{16 - x} = \lim_{x \rightarrow 16} (4 + \sqrt{x}) \end{aligned}$$

Substitute 16 into the limit for  $x$ .

$$\lim_{x \rightarrow 16} (4 + \sqrt{x}) = 4 + \sqrt{16} = 4 + 4 = 8$$

**Answer:**  $\lim_{x \rightarrow 16} \frac{16 - x}{4 - \sqrt{x}} = 8$

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9. Evaluate this limit.

$$\lim_{x \rightarrow -4} \frac{8x + 32}{x^2 + 5x + 4}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute  $-4$  into the limit for  $x$ .

$$\lim_{x \rightarrow -4} \frac{8x + 32}{x^2 + 5x + 4} = \frac{8(-4) + 32}{(-4)^2 + 5(-4) + 4} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow -4} \frac{8x + 32}{x^2 + 5x + 4} = \lim_{x \rightarrow -4} \frac{8(x + 4)}{(x + 1)(x + 4)} = \lim_{x \rightarrow -4} \frac{8}{x + 1}$$

Substitute  $-4$  into the limit for  $x$ .

$$\lim_{x \rightarrow -4} \frac{8}{x + 1} = \frac{8}{-4 + 1} = \frac{8}{-3} = -\frac{8}{3}$$

**Answer:**  $\lim_{x \rightarrow -4} \frac{8x+32}{x^2+5x+4} = -\frac{8}{3}$

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10. Evaluate this limit.

$$\lim_{x \rightarrow -3} \frac{9x + 27}{x^2 + 5x + 6}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute  $-3$  into the limit for  $x$ .

$$\lim_{x \rightarrow -3} \frac{9x + 27}{x^2 + 5x + 6} = \frac{9(-3) + 27}{(-3)^2 + 5(-3) + 6} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow -3} \frac{9x + 27}{x^2 + 5x + 6} = \lim_{x \rightarrow -3} \frac{9(x + 3)}{(x + 3)(x + 2)} = \lim_{x \rightarrow -3} \frac{9}{x + 2}$$

Substitute  $-3$  into the limit for  $x$ .

$$\lim_{x \rightarrow -3} \frac{9}{x + 2} = \frac{9}{-3 + 2} = \frac{9}{-1} = -9$$

**Answer:**  $\lim_{x \rightarrow -3} \frac{9x + 27}{x^2 + 5x + 6} = -9$

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11. Evaluate this limit.

$$\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{-4x^2 - 16x - 12}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute  $-3$  into the limit for  $x$ .

$$\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{-4x^2 - 16x - 12} = \frac{(-3)^2 + (-3) - 6}{-4(-3)^2 - 16(-3) - 12} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\begin{aligned} \lim_{x \rightarrow -3} \frac{x^2 + x - 6}{-4x^2 - 16x - 12} &= \lim_{x \rightarrow -3} \frac{(x + 3)(x - 2)}{-4(x^2 + 4x + 3)} \\ &= \lim_{x \rightarrow -3} \frac{(x + 3)(x - 2)}{-4(x + 1)(x + 3)} = \lim_{x \rightarrow -3} \frac{x - 2}{-4(x + 1)} \end{aligned}$$

Substitute  $-3$  into the limit for  $x$ .

$$= \lim_{x \rightarrow -3} \frac{x - 2}{-4(x + 1)} = \frac{-3 - 2}{-4(-3 + 1)} = -\frac{5}{8}$$

**Answer:**  $\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{-4x^2 - 16x - 12} = -\frac{5}{8}$

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12. Evaluate this limit.

$$\lim_{x \rightarrow 3} \frac{x^3 - 9x}{3x^2 - 6x - 9}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute 3 into the limit for  $x$ .

$$\lim_{x \rightarrow 3} \frac{x^3 - 9x}{3x^2 - 6x - 9} = \frac{3^3 - 9(3)}{3(3)^2 - 6(3) - 9} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\begin{aligned} \lim_{x \rightarrow 3} \frac{x^3 - 9x}{3x^2 - 6x - 9} &= \lim_{x \rightarrow 3} \frac{x(x^2 - 9)}{3(x^2 - 2x - 3)} \\ &= \lim_{x \rightarrow 3} \frac{x(x + 3)(x - 3)}{3(x - 3)(x + 1)} = \lim_{x \rightarrow 3} \frac{x(x + 3)}{3(x + 1)} \end{aligned}$$

Substitute 3 into the limit for  $x$ .

$$\lim_{x \rightarrow 3} \frac{x(x + 3)}{3(x + 1)} = \frac{3(3 + 3)}{3(3 + 1)} = \frac{18}{12} = \frac{3}{2}$$

**Answer:**  $\lim_{x \rightarrow 3} \frac{x^3 - 9x}{3x^2 - 6x - 9} = \frac{3}{2}$

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13. Evaluate this limit.

$$\lim_{x \rightarrow 25} \frac{5 - \sqrt{x}}{25 - x}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute 25 into the limit for  $x$ .

$$\lim_{x \rightarrow 25} \frac{5 - \sqrt{x}}{25 - x} = \frac{5 - \sqrt{25}}{25 - 25} = \frac{5 - 5}{25 - 25} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, rationalize the numerator and simplify the limit.

$$\begin{aligned} \lim_{x \rightarrow 25} \frac{5 - \sqrt{x}}{25 - x} &= \lim_{x \rightarrow 25} \frac{5 - \sqrt{x}}{25 - x} \cdot \frac{5 + \sqrt{x}}{5 + \sqrt{x}} \\ &= \lim_{x \rightarrow 25} \frac{25 - x}{(25 - x)(5 + \sqrt{x})} = \lim_{x \rightarrow 25} \frac{1}{5 + \sqrt{x}} \end{aligned}$$

Substitute 25 into the limit for  $x$ .

$$\lim_{x \rightarrow 25} \frac{1}{5 + \sqrt{x}} = \frac{1}{5 + \sqrt{25}} = \frac{1}{5 + 5} = \frac{1}{10}$$

**Answer:**  $\lim_{x \rightarrow 25} \frac{5 - \sqrt{x}}{25 - x} = \frac{1}{10}$

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14. Evaluate this limit.

$$\lim_{x \rightarrow -2} \frac{2x^2 + 10x + 12}{x^2 + 3x + 2}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute  $-2$  into the limit for  $x$ .

$$\lim_{x \rightarrow -2} \frac{2x^2 + 10x + 12}{x^2 + 3x + 2} = \frac{2(-2)^2 + 10(-2) + 12}{(-2)^2 + 3(-2) + 2} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\begin{aligned} \lim_{x \rightarrow -2} \frac{2x^2 + 10x + 12}{x^2 + 3x + 2} &= \lim_{x \rightarrow -2} \frac{2(x^2 + 5x + 6)}{x^2 + 3x + 2} \\ \lim_{x \rightarrow -2} \frac{2(x + 3)(x + 2)}{(x + 2)(x + 1)} &= \lim_{x \rightarrow -2} \frac{2(x + 3)}{x + 1} \end{aligned}$$

Substitute  $-2$  into the limit for  $x$ .

$$\lim_{x \rightarrow -2} \frac{2(x + 3)}{x + 1} = \frac{2(-2 + 3)}{-2 + 1} = \frac{2}{-1} = -2$$

Answer:  $\lim_{x \rightarrow -2} \frac{2x^2 + 10x + 12}{x^2 + 3x + 2} = -2$

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15. Evaluate this limit.

$$\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x^2 + 8x + 15}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute  $-3$  into the limit for  $x$ .

$$\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x^2 + 8x + 15} = \frac{(-3)^2 + (-3) - 6}{(-3)^2 + 8(-3) + 15} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x^2 + 8x + 15} = \lim_{x \rightarrow -3} \frac{(x + 3)(x - 2)}{(x + 3)(x + 5)} = \lim_{x \rightarrow -3} \frac{x - 2}{x + 5}$$

Substitute  $-3$  into the limit for  $x$ .

$$\lim_{x \rightarrow -3} \frac{x - 2}{x + 5} = \frac{-3 - 2}{-3 + 5} = -\frac{5}{2}$$

**Answer:**  $\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x^2 + 8x + 15} = -\frac{5}{2}$

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16. Evaluate this limit.

$$\lim_{x \rightarrow -4} \frac{x^2 + 5x + 4}{x^2 + 9x + 20}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute  $-4$  into the limit for  $x$ .

$$\lim_{x \rightarrow -4} \frac{x^2 + 5x + 4}{x^2 + 9x + 20} = \frac{(-4)^2 + 5(-4) + 4}{(-4)^2 + 9(-4) + 20} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow -4} \frac{x^2 + 5x + 4}{x^2 + 9x + 20} = \lim_{x \rightarrow -4} \frac{(x + 4)(x + 1)}{(x + 5)(x + 4)} = \lim_{x \rightarrow -4} \frac{x + 1}{x + 5}$$

Substitute  $-4$  into the limit for  $x$ .

$$\lim_{x \rightarrow -4} \frac{x + 1}{x + 5} = \frac{-4 + 1}{-4 + 5} = -3$$

**Answer:**  $\lim_{x \rightarrow -4} \frac{x^2 + 5x + 4}{x^2 + 9x + 20} = -3$

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17. Evaluate this limit.

$$\lim_{x \rightarrow 5} \frac{2x^2 - 4x - 30}{3x^2 - 18x + 15}$$

First, attempt to evaluate the limit using direct substitution. Substitute 5 into the limit for  $x$ .

$$\lim_{x \rightarrow 5} \frac{2x^2 - 4x - 30}{3x^2 - 18x + 15} = \frac{2(5)^2 - 4(5) - 30}{3(5)^2 - 18(5) + 15} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\begin{aligned} \lim_{x \rightarrow 5} \frac{2x^2 - 4x - 30}{3x^2 - 18x + 15} &= \lim_{x \rightarrow 5} \frac{2(x^2 - 2x - 15)}{3(x^2 - 6x + 5)} \\ &= \lim_{x \rightarrow 5} \frac{2(x - 5)(x + 3)}{3(x - 5)(x - 1)} = \lim_{x \rightarrow 5} \frac{2(x + 3)}{3(x - 1)} \end{aligned}$$

Substitute 5 into the limit for  $x$ .

$$= \lim_{x \rightarrow 5} \frac{2(x + 3)}{3(x - 1)} = \frac{2(5 + 3)}{3(5 - 1)} = \frac{2(8)}{3(4)} = \frac{4}{3}$$

**Answer:**  $\lim_{x \rightarrow 5} \frac{2x^2 - 4x - 30}{3x^2 - 18x + 15} = \frac{4}{3}$

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18. Evaluate this limit.

$$\lim_{x \rightarrow 0} \frac{16x^3 + 80x^2}{40x^3 - 80x^2}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute 0 into the limit for  $x$ .

$$\lim_{x \rightarrow 0} \frac{16x^3 + 80x^2}{40x^3 - 80x^2} = \frac{16(0)^3 + 80(0)^2}{40(0)^3 - 80(0)^2} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow 0} \frac{16x^3 + 80x^2}{40x^3 - 80x^2} = \lim_{x \rightarrow 0} \frac{16x^2(x + 5)}{40x^2(x - 2)} = \lim_{x \rightarrow 0} \frac{2(x + 5)}{5(x - 2)}$$

Substitute 0 into the limit for  $x$ .

$$\lim_{x \rightarrow 0} \frac{2(x + 5)}{5(x - 2)} = \frac{2(0 + 5)}{5(0 - 2)} = \frac{10}{-10} = -1$$

**Answer:**  $\lim_{x \rightarrow 0} \frac{16x^3 + 80x^2}{40x^3 - 80x^2} = -1$

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19. Evaluate this limit.

$$\lim_{x \rightarrow -4} \frac{x^3 + 64}{x + 4}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute  $-4$  into the limit for  $x$ .

$$\lim_{x \rightarrow -4} \frac{x^3 + 64}{x + 4} = \frac{(-4)^3 + 64}{-4 + 4} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow -4} \frac{x^3 + 64}{x + 4} = \lim_{x \rightarrow -4} \frac{(x + 4)(x^2 - 4x + 16)}{x + 4} = \lim_{x \rightarrow -4} (x^2 - 4x + 16)$$

Substitute  $-4$  into the limit for  $x$ .

$$\lim_{x \rightarrow -4} (x^2 - 4x + 16) = (-4)^2 - 4(-4) + 16 = 48$$

**Answer:**  $\lim_{x \rightarrow -4} \frac{x^3 + 64}{x + 4} = 48$

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20. Evaluate this limit.

$$\lim_{x \rightarrow -1} \frac{x^3 + x^2 - 9x - 9}{x^3 + x^2 - 4x - 4}$$

Solution:

First, attempt to evaluate the limit using direct substitution. Substitute  $-1$  into the limit for  $x$ .

$$\lim_{x \rightarrow -1} \frac{x^3 + x^2 - 9x - 9}{x^3 + x^2 - 4x - 4} = \frac{(-1)^3 + (-1)^2 - 9(-1) - 9}{(-1)^3 + (-1)^2 - 4(-1) - 4} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\begin{aligned} \lim_{x \rightarrow -1} \frac{x^3 + x^2 - 9x - 9}{x^3 + x^2 - 4x - 4} &= \lim_{x \rightarrow -1} \frac{(x + 1)(x^2 - 9)}{(x + 1)(x^2 - 4)} \\ &= \lim_{x \rightarrow -1} \frac{x^2 - 9}{x^2 - 4} \end{aligned}$$

Substitute  $-1$  into the limit for  $x$ .

$$\lim_{x \rightarrow -1} \frac{x^2 - 9}{x^2 - 4} = \frac{(-1)^2 - 9}{(-1)^2 - 4} = \frac{1 - 9}{1 - 4} = \frac{-8}{-3} = \frac{8}{3}$$

**Answer:**  $\lim_{x \rightarrow -1} \frac{x^3 + x^2 - 9x - 9}{x^3 + x^2 - 4x - 4} = \frac{8}{3}$

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