

## Evaluating Limits

Evaluate each limit.

$$1) \lim_{x \rightarrow -1} 5$$

$$5$$

$$2) \lim_{x \rightarrow -\frac{5}{2}} (-x + 2)$$

$$\frac{9}{2}$$

$$3) \lim_{x \rightarrow 2} (x^3 - x^2 - 4)$$

$$0$$

$$4) \lim_{x \rightarrow 1} \left( -\frac{x^2}{2} + 2x + 4 \right)$$

$$\frac{11}{2}$$

$$5) \lim_{x \rightarrow 3} -\sqrt{x+3}$$

$$-\sqrt{6}$$

$$6) \lim_{x \rightarrow \frac{3}{2}} -\sqrt{2x+4}$$

$$-\sqrt{7}$$

$$7) \lim_{x \rightarrow 1} -\frac{x-4}{x^2-6x+8}$$

$$1$$

$$8) \lim_{x \rightarrow \frac{3}{2}} \frac{-x-3}{x^2+x+1}$$

$$-\frac{18}{19}$$

$$9) \lim_{x \rightarrow \pi} \sin(x)$$

$$0$$

$$10) \lim_{x \rightarrow \frac{3\pi}{4}} 2\cos(x)$$

$$-\sqrt{2}$$

## Critical thinking questions:

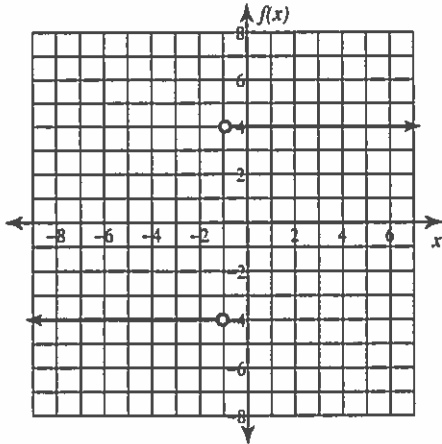
11) Give an example of a limit that evaluates to 4.  
Many answers. Ex:  $\lim_{x \rightarrow 4} x$

12) Give an example of a limit of a quadratic function where the limit evaluates to 9.  
Many answers. Ex:  $\lim_{x \rightarrow 3} x^2$

# Evaluating Limits

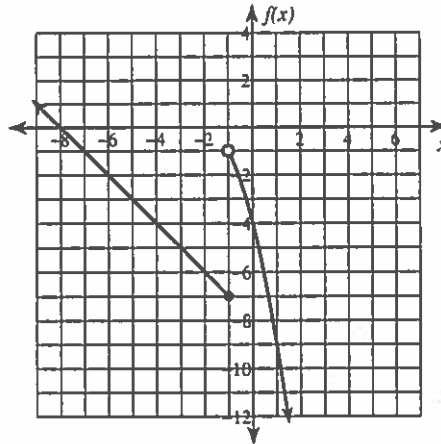
Evaluate each limit.

1)  $\lim_{x \rightarrow -1^+} \frac{4x + 4}{|x + 1|}$



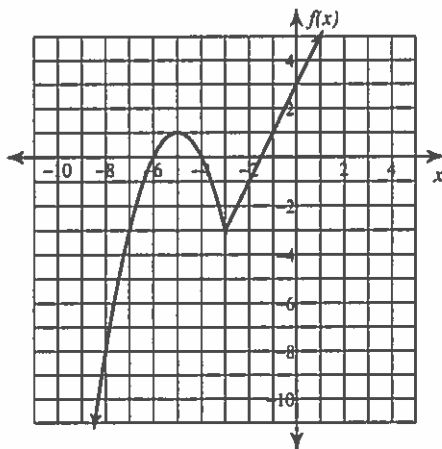
4

2)  $\lim_{x \rightarrow -1^-} f(x), f(x) = \begin{cases} -x - 8, & x \leq -1 \\ -x^2 - 4x - 4, & x > -1 \end{cases}$



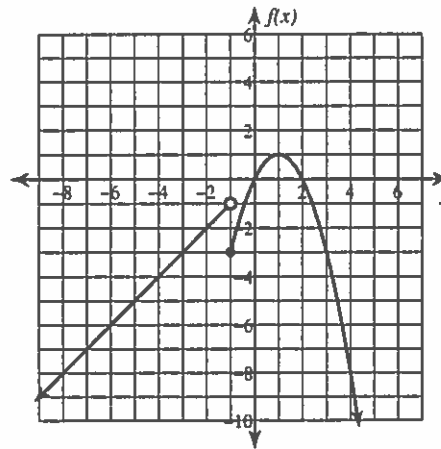
-7

3)  $\lim_{x \rightarrow -3} f(x), f(x) = \begin{cases} -x^2 - 10x - 24, & x \leq -3 \\ 2x + 3, & x > -3 \end{cases}$



-3

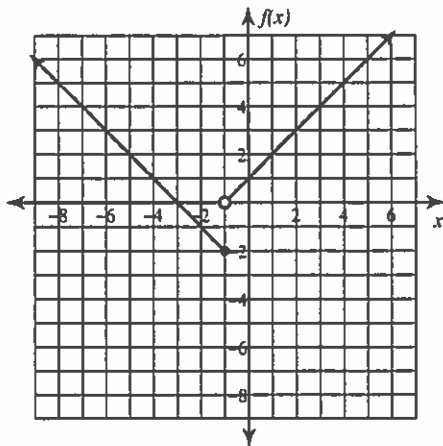
4)  $\lim_{x \rightarrow -1} f(x), f(x) = \begin{cases} x, & x < -1 \\ -x^2 + 2x, & x \geq -1 \end{cases}$



Does not exist.

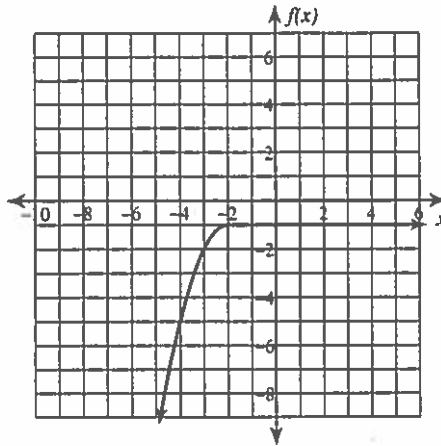
Evaluate each limit. You may use the provided graph to sketch the function.

$$5) \lim_{x \rightarrow -1^-} f(x), f(x) = \begin{cases} -x - 3, & x \leq -1 \\ x + 1, & x > -1 \end{cases}$$



-2

$$6) \lim_{x \rightarrow -2} f(x), f(x) = \begin{cases} -x^2 - 4x - 5, & x \leq -2 \\ -1, & x > -2 \end{cases}$$



-1

Evaluate each limit.

$$7) \lim_{x \rightarrow 0^+} f(x), f(x) = \begin{cases} 1, & x \leq 0 \\ -x^2 + 4x - 3, & x > 0 \end{cases}$$

-3

$$8) \lim_{x \rightarrow 0^-} \frac{|x|}{x}$$

-1

$$9) \lim_{x \rightarrow 0^+} [-2x + 1]$$

0

$$10) \lim_{x \rightarrow 1} f(x), f(x) = \begin{cases} \frac{x}{2} + \frac{9}{2}, & x < 1 \\ x^2 - 6x + 10, & x \geq 1 \end{cases}$$

5

$$11) \lim_{x \rightarrow -1} \frac{3|x+1|}{x+1}$$

Does not exist.

$$12) \lim_{x \rightarrow -2} f(x), f(x) = \begin{cases} x^2, & x \leq -2 \\ -\frac{x}{2} + 3, & x > -2 \end{cases}$$

4

Critical thinking questions:

13) Give an example of a two-sided limit of a piecewise function where the limit does not exist.

Many answers. Ex:  $\lim_{x \rightarrow 1} f(x), f(x) = \begin{cases} 0, & x < 1 \\ x, & x > 1 \end{cases}$

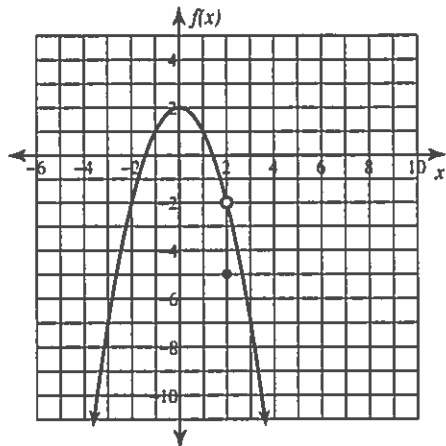
14) Given an example of a two-sided limit of a function with an absolute value where the limit does not exist.

Many answers. Ex:  $\lim_{x \rightarrow 0} \frac{|x|}{x}$

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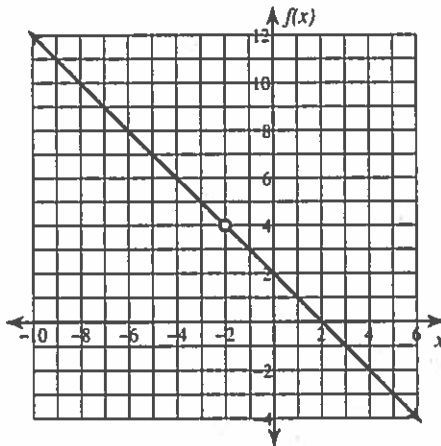
Evaluate each limit.

1)  $\lim_{x \rightarrow 2} f(x), f(x) = \begin{cases} -x^2 + 2, & x \neq 2 \\ -5, & x = 2 \end{cases}$



-2

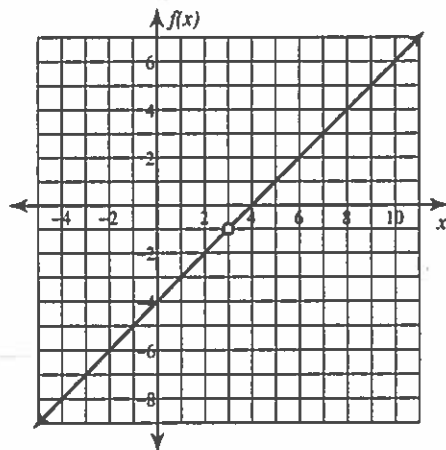
2)  $\lim_{x \rightarrow -2} -\frac{x^2 - 4}{x + 2}$



4

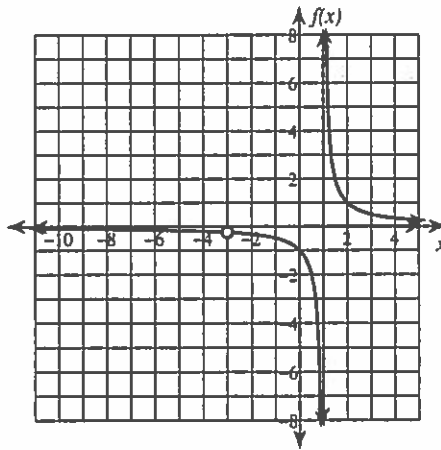
Evaluate each limit. You may use the provided graph to sketch the function.

3)  $\lim_{x \rightarrow 3} \frac{x^2 - 7x + 12}{x - 3}$



-1

4)  $\lim_{x \rightarrow -3} \frac{x + 3}{x^2 + 2x - 3}$

 $-\frac{1}{4}$ 

Evaluate each limit.

5)  $\lim_{x \rightarrow 0} f(x), f(x) = \begin{cases} x + 1, & x \neq 0 \\ 2, & x = 0 \end{cases}$

1

6)  $\lim_{x \rightarrow 3} f(x), f(x) = \begin{cases} 2 + \frac{x}{2}, & x \neq 3 \\ 2, & x = 3 \end{cases}$

 $\frac{7}{2}$

$$7) \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$$

-2

$$8) \lim_{x \rightarrow 5} \frac{x^2 - 5x}{x - 5}$$

-5

$$9) \lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x - 2}$$

-3

$$10) \lim_{x \rightarrow -5} \frac{x^2 + 3x - 10}{x + 5}$$

-7

$$11) \lim_{x \rightarrow 0} \frac{\frac{1}{-4+x} + \frac{1}{4}}{x}$$

-\frac{1}{16}

$$12) \lim_{x \rightarrow -3} \frac{x}{\frac{1}{3+x} - \frac{1}{3}}$$

0

$$13) \lim_{x \rightarrow 5} \frac{x - 5}{\sqrt{x + 4} - 3}$$

6

$$14) \lim_{x \rightarrow 3} \frac{\sqrt{x + 6} - 3}{x - 3}$$

\frac{1}{6}

### Critical thinking questions:

15) Give an example of a limit of a rational function where the limit at -1 exists, but the rational function is undefined at -1.

Many answers. Ex:  $\lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1}$

16) Give two values of  $a$  where the limit cannot be solved using direct evaluation. Give one value of  $a$  where the limit can be solved using direct evaluation.

$$\lim_{x \rightarrow a} \frac{x}{\frac{1}{-2+x} + \frac{1}{2}}$$

No direct eval:  $a=0,2$  Direct eval:  $a=\text{any other number}$