

Differentiation Rules, with Tables

For each problem, you are given a table containing some values of differentiable functions $f(x)$, $g(x)$ and their derivatives. Use the table data and the rules of differentiation to solve each problem.

1)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	1	1	2
2	3	0	3	0
3	2	-1	1	-2

Given $h(x) = f(x) + g(x)$, find $h'(1)$

$$h'(x) = f'(x) + g'(x)$$

$$h'(1) = f'(1) + g'(1)$$

$$= 1 + 2$$

$$= \boxed{3}$$

2)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	-1	1	1
2	1	$\frac{1}{2}$	2	1
3	3	2	3	1

Given $h(x) = f(x) - g(x)$, find $h'(2)$

$$h'(x) = f'(x) - g'(x)$$

$$h'(2) = f'(2) - g'(2)$$

$$= \frac{1}{2} - 1$$

$$= \boxed{-\frac{1}{2}}$$

3)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	-1	1	2
2	1	$\frac{1}{2}$	3	0
3	3	2	1	-2

Given $h(x) = f(x) \cdot g(x)$, find $h'(3)$

Product Rule

$$h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$= (2)(1) + (3)(-2)$$

$$= 2 - 6$$

$$= \boxed{-4}$$

4)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	-1	2	-1
2	2	-1	1	0
3	1	-1	2	1

Given $h(x) = \frac{f(x)}{g(x)}$, find $h'(3)$

Quotient Rule

$$h'(x) = \frac{f'(x)g(x) - [f(x) \cdot g'(x)]}{[g(x)]^2}$$

$$= \frac{(-1)(2) - [(1)(1)]}{2^2}$$

$$= \frac{-2 - 1}{4} = \boxed{-\frac{3}{4}}$$

x	f(x)	f'(x)	g(x)	g'(x)
1	5	-1	1	2
2	4	-1	3	$\frac{3}{2}$
3	3	-1	4	1
4	2	-1	5	1
5	1	0	6	$-\frac{1}{2}$
6	2	1	4	-2

Part 1) Given $h_1(x) = f(x) + g(x)$, find $h_1'(2)$

Part 2) Given $h_2(x) = f(x) - g(x)$, find $h_2'(3)$

Part 3) Given $h_3(x) = f(x) \cdot g(x)$, find $h_3'(4)$

Part 4) Given $h_4(x) = \frac{f(x)}{g(x)}$, find $h_4'(2)$

Part 5) Given $h_5(x) = (f(x))^2$, find $h_5'(2)$

Part 6) Given $h_6(x) = f(g(x))$, find $h_6'(6)$

} Chain Rule

$$\textcircled{1} \quad h_1'(2) = f'(2) + g'(2) = (-1) + \frac{3}{2} = \boxed{\frac{1}{2}}$$

$$\textcircled{2} \quad h_2'(3) = f'(3) - g'(3) = (-1) - 1 = \boxed{-2}$$

$$\textcircled{3} \quad h_3'(4) = f'(4)g(4) + f(4)g'(4) = (-1)(5) + (2)(1) = -5 + 2 = \boxed{-3}$$

$$\textcircled{4} \quad h_4'(2) = \frac{f'(2)g(2) - f(2)g'(2)}{[g(2)]^2} = \frac{(-1)(3) - [(4)(\frac{3}{2})]}{3^2}$$

$$= \frac{-3 - 6}{9} = \frac{-9}{9} = \boxed{-1}$$

$$\textcircled{5} \quad h_5'(x) = 2 \cdot f(x)^{2-1} \cdot f'(x) = 2f(2) \cdot f'(2) = (2)(4)(-1) = \boxed{-8}$$

$$\textcircled{6} \quad h_6'(x) = f'(g(x)) \cdot g'(x) = f'(g(6)) \cdot g'(6)$$

$$= f'(4) \cdot g'(6) = (-1)(-2) = \boxed{2}$$