DIFFERENTIAL EQUATIONS

> An equation involving a derivative is called a differential equation. The order of a differential equation is given by the order of the highest derivative in the equation.

Example 1: Find all functions y that satisfy: $dy/dx = \csc^2 x + 3x^2 + 5x - 6$

Solution:

Note: Without any additional information we can only find the **general solution to the differential equation** as the one above. The general solution is the family of functions that are the antiderivative of the expression that equals to the dy/dx.

Example 2: Find the general solution to $dy/dz = -\cos z + e^z - 5z^4 - 1$

> If the general solution to a fist-order differential equation is continuous, we need only one piece of information to find a unique solution to the differential equation.
> This piece of information is the **value of the function (= general solution) at a single point (that is, the coordinates of a single point that lies on the graph of the function that represents the general solution. This piece of information is called the INITIAL CONDITION.**
> A differential equation with initial condition is referred to as **an initial value problem.**
> The unique solution to the differential equation is called the **particular solution.**

Example 3: Find the particular solution of $dy/dx = e^x - 6x^2$ whose graph passes through the point (1,0).

- General solution:
- Particular solution:

Example 4: Find the particular solution of $dy/dx = \cos x + x^{-1} + 3$ that passes through the point $(\pi/2, 7)$

Example 5: Find the particular solution of $dy/dx = \csc^2 x - 3x^2$ whose graph passes through the point $(\pi/2, 3)$

➢ **Note, due to the fact that cotx has periodic infinite discontinuity at every multiple of π the domain must be restricted to the interval where the general solution is continuous.**

Example 6: Use the FTC part 1 to find the particular solution to $dy/dx = e^{-x^2}$ for which $f(7) = 3$

Solution: $\int_7^x e^{-t^2}\, dt + 3$

Example 7: Use the FTC part 1 to find the particular solution to $dy/dx = \cos^2 4x$ for which $f(3) = 8$

HW: p327 #1-12