C12

Connecting f’ and f’’ with the graph of f

Theorem: First Derivative Test for Local Extrema

The following test applies to a continuous function f(x).

At a critical point c:

1. If f’ changes sign from positive to negative at c (f’ is greater than zero for all x smaller than c and f’ is less than zero for all x greater than c), then f has a local maximum value at c.

2. If f’ changes sign from negative to positive at c (f’ is less than zero for x less than c and f’ is greater than zero for x greater than c), then f has a local minimum value at c.

3. If f’ does not change sign at c (f’ has the same sign on both sides of c), then f has no local extreme value at c.

At a left endpoint a:

If f’ is less than zero (f’ greater than zero) for x greater than a, then f has a local maximum (minimum) value at a.

At a right endpoint b:

If f’ is less than zero (f’ is greater than zero) for x smaller than b, then f has a local minimum (maximum) value at b.

Example: Use the first derivative test to find the local extrema. Identify any absolute extrema if they exist given a function f(x) $=4x^3 - 5x + x$.

CONCAVITY

Definition: The graph of a differentiable function y=f(x) is

a) **Concave up** on an open interval I if y' is increasing on I.

b) **Concave down** on an open interval I if y' is decreasing on I.

Concavity test:

The twice-differentiable function y=f(x) is

a) **Concave up** on any interval where y'' is positive.

b) **Concave down** on any interval where y'' is negative.

Definition: Point of Inflection

A point where the graph of a function has a tangent line **and** where the concavity changes is point of inflection.

Theorem: Second Derivative Test for Local Extrema

1. If f'(c) = 0 and f''(c) is negative, then f has a local maximum at x=c.

2. If f'(c) = 0 and f''(c) is positive, then f has a local minimum at x=c.

HW: p 215-217 #1-28 – do at least 14 questions and #45-50