DERIVATIVE TESTS AND CURVE SKETCHING

Note: It is always helpful to sketch the function even when you are not required to do so.

1. Clearly identify intervals on which the given function is increasing and on which it is decreasing.

a) $f(x) = x^5 - 3x + 15$

b) $g(x) = \sqrt{9 - x^2}$

c) $h(x) = \dfrac{1}{\log x}$

d) $f(x) = \dfrac{1-x}{x^2-4}$

2. Clearly identify intervals where the given function is concave up and where it is concave down.

a) $f(x) = x^4 - 2x^3 + x - 6$

b) $g(x) = \dfrac{x^2}{x-3}$

c) $h(x) = \sqrt{x^2 - 9}$

*d) $f(x) = \dfrac{x - 3x^2}{x^2 - 2}$

3. Sketch a graph of the following:

Function without extrema.	Function with a local maximum but without an absolute maximum.	Function with two local maxima and one absolute minimum.	Function with no absolute extrema.
Function with only one inflection point at x = 5.	Function that is concave up on its entire domain.	Function that is increasing on (-5,0) and decreasing on (0,10).	Function whose first derivative is 2.

4. Identify <u>coordinates</u> of all critical points and identify why each point is a critical point. Support your statements. **Identify whether a local/absolute extreme point, inflection point, discontinuity, corner, cusp or vertical tangent occur at the critical point.**

a) $f(x) = x^2 - 3$

b) $y = \sqrt[3]{x - 7}$

c) $g(x) = \sqrt{x + 5}$

d) $h(x) = 4x^4 - 18x^2 + 18$

5. Sketch two different functions that have inflection points where the first derivative exists and is NOT zero.

6. Sketch two graphs of functions that change concavity without having an inflection point.

7. Use the second derivative test to identify any extreme values.

a) $f(x) = \frac{4}{3}x^3 + 2x^2 - 3x$

b) $y = \dfrac{x^3 - 2}{x^2}$

8. State the Mean Value Theorem. Include a labeled diagram in your definition. Give an example of a quadratic function and show that the MVT holds for the function on the interval [-6,10]. Clearly identify the point at which the function has a tangent with the same slope as the secant line specified by the interval above. **Include a labeled diagram.**