

Assignment 5.4 2nd Derivative Test

Date _____ Period _____

For each problem, find all points of relative minima and maxima.

1) $f(x) = -x^3 + 3x^2 - 2$

2) $f(x) = x^4 - 2x^2 - 1$

3) $f(x) = -x^3 + 6x^2 - 9x + 7$

4) $f(x) = x^3 - 3x^2 - 3$

5) $f(x) = -x^3 - 9x^2 - 24x - 21$

6) $f(x) = -x^4 + 2x^2 + 4$

7) $f(x) = x^3 - 3x^2 + 5$

8) $f(x) = x^4 - 2x^2 + 4$

For each problem, find the x-coordinates of all points of inflection and find the open intervals where the function is concave up and concave down.

9) $f(x) = -x^3 + 4x^2 - 2$

10) $f(x) = -x^3 + x^2 + 5x - 3$

11) $f(x) = -x^4 + x^3 + 3x^2 - 3$

12) $f(x) = x^4 - 3x^3 + 5x + 1$

13) $f(x) = \frac{2x}{x+1}$

14) $f(x) = \frac{2}{x+1}$

For each problem, find all points of relative minima and maxima.

15) $f(x) = -x^4 + 2x^2 + 3$

For each problem, find the open intervals where the function is increasing and decreasing.

16) $f(x) = -x^4 + 4x^2$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y .

17) $5x + y^3 = 3y$

18) $2y^2 = 2x^3 + 2y$

19) $5y = 5x^2 - 3y^3$

20) $2y = 2x^2 - y^3$

21) $-4y^3 + 2xy = 5x^3$

22) $4x^2 - 2y^2 = 2x^3y^3$

23) $2x = 2xy + 1$

24) $2 = 3x - 2x^3y^2$

For each problem, find the indicated derivative with respect to x .

25) $f(x) = -x^4 + x^3 + x$ Find f''

26) $f(x) = -3x^3$ Find $f^{(4)}$

27) $f(x) = -x^5 - x^4 + 3x^2$ Find f'''

28) $f(x) = -4x^2$ Find f''

Differentiate each function with respect to x .

29) $f(x) = (-5x^3 - 3)(-3x^3 + 4)$

30) $f(x) = \frac{3}{x^2 + 4}$

Answers to Assignment 5.4 2nd Derivative Test (ID: 1)

- 1) Relative minimum: $(0, -2)$ 2) Relative minima: $(-1, -2), (1, -2)$
 Relative maximum: $(2, 2)$ Relative maximum: $(0, -1)$
 3) Relative minimum: $(1, 3)$ 4) Relative minimum: $(2, -7)$ 5) Relative minimum: $(-4, -5)$
 Relative maximum: $(3, 7)$ Relative maximum: $(0, -3)$ Relative maximum: $(-2, -1)$
 6) Relative minimum: $(0, 4)$ 7) Relative minimum: $(2, 1)$
 Relative maxima: $(-1, 5), (1, 5)$ Relative maximum: $(0, 5)$
 8) Relative minima: $(-1, 3), (1, 3)$ 9) Inflection point at: $x = \frac{4}{3}$
 Relative maximum: $(0, 4)$ Concave up: $(-\infty, \frac{4}{3})$ Concave down: $(\frac{4}{3}, \infty)$

10) Inflection point at: $x = \frac{1}{3}$
 Concave up: $(-\infty, \frac{1}{3})$ Concave down: $(\frac{1}{3}, \infty)$

11) Inflection points at: $x = -\frac{1}{2}, 1$
 Concave up: $(-\frac{1}{2}, 1)$ Concave down: $(-\infty, -\frac{1}{2}), (1, \infty)$

12) Inflection points at: $x = 0, \frac{3}{2}$
 Concave up: $(-\infty, 0), (\frac{3}{2}, \infty)$ Concave down: $(0, \frac{3}{2})$

13) No inflection points exist.
 Concave up: $(-\infty, -1)$ Concave down: $(-1, \infty)$

14) No inflection points exist.
 Concave up: $(-1, \infty)$ Concave down: $(-\infty, -1)$

15) Relative minimum: $(0, 3)$
 Relative maxima: $(-1, 4), (1, 4)$

16) Increasing: $(-\infty, -\sqrt{2}), (0, \sqrt{2})$ Decreasing: $(-\sqrt{2}, 0), (\sqrt{2}, \infty)$

17) $\frac{dy}{dx} = -\frac{5}{3y^2 - 3}$

18) $\frac{dy}{dx} = \frac{3x^2}{2y - 1}$

19) $\frac{dy}{dx} = \frac{10x}{5 + 9y^2}$

20) $\frac{dy}{dx} = \frac{4x}{2 + 3y^2}$

21) $\frac{dy}{dx} = \frac{15x^2 - 2y}{-12y^2 + 2x}$

22) $\frac{dy}{dx} = \frac{3x^2y^3 - 4x}{-2y - 3y^2x^3}$

23) $\frac{dy}{dx} = \frac{-y + 1}{x}$

24) $\frac{dy}{dx} = \frac{3 - 6x^2y^2}{4x^3y}$

25) $f''(x) = -12x^2 + 6x$

26) $f^{(4)}(x) = 0$

27) $f'''(x) = -60x^2 - 24x$

28) $f''(x) = -8$

29) $f'(x) = (-5x^3 - 3) \cdot -9x^2 + (-3x^3 + 4) \cdot -15x^2$
 $= 90x^5 - 33x^2$

30) $f'(x) = -\frac{3 \cdot 2x}{(x^2 + 4)^2}$
 $= -\frac{6x}{x^4 + 8x^2 + 16}$