Continuity

KEY

#1. $f(x) = \frac{x}{x^2+3x} = \frac{\cancel{x}}{\cancel{x}(x+3)} = \frac{1}{x+3}$ NPVs: $x(x+3)=0$

$\boxed{x=0}$ $x+3=0$ $\boxed{x=-3}$

- $f(x)$ is not continuous at $x=0$ because $0 \notin D_f$.
- $f(x)$ is not continuous at $x=-3$ because $-3 \notin D_f$.

- $x=0$ gives a PD = removable discontinuity
- $x=-3$ gives a VA: $x=-3$; infinite = non-removable discontinuity.

#2 $f(x) = \begin{cases} -2 ; & x \le 3 \\ x-5 ; & x > 3 \end{cases}$

- $f(x)$ is defined at $x=3$
- $\lim_{x \to 3} f(x) = \underline{\underline{-2}}$
- $\lim_{x \to 3^+} f(x) = 3-5 = -2$
- $\lim_{x \to 3^-} f(x) = -2$
- $f(3) = -2$

$\lim_{x \to 3} f(x) = f(3) = -2$ ✓

$\therefore f(x)$ is continuous at $x=3$.

#3 $f(x) = \dfrac{x+1}{x^2+2x+2}$

NPVs: $x^2+2x+2=0$

$$x = \frac{-2 \pm \sqrt{2^2-4(1)(2)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4-8}}{2} \Rightarrow \text{no } \mathbb{R} \text{ sol}$$

$\Rightarrow$ no NPVs

- $-3 \in D_f$ ✓

- $\displaystyle\lim_{x \to -3} \frac{x+1}{x^2+2x+2} = \frac{-2}{5}$

- $f(-3) = \frac{-2}{5}$

$\Big\}\ \displaystyle\lim_{x \to -3} f(x) = f(-3) = -2/5$ ✓

$\therefore f(x)$ is continuous at $x = -3$.

#4 $f(x) = \dfrac{x+2}{x^2+4} = \dfrac{\cancel{(x+2)}}{\cancel{(x+2)}(x-2)} = \dfrac{1}{x-2}$

NPVs:

$(x+2)(x-2) = 0$

$\boxed{x=-2}$ $\boxed{x=2}$

- $-2 \notin D_f$
- $2 \notin D_f$

$\Big\}\ \therefore f(x)$ is not continuous at $x=-2$ and $x=2$.

- $x=-2$ gives a PD $\rightarrow$ removable disc.
- $x=2$ gives a VA $\Rightarrow$ infinite = non-removable disc.

#5 $f(x) = \dfrac{x^2}{x+1}$ NPV: $x+1=0$

$\boxed{x=-1}$

- $-1 \notin D_f$: $f(x)$ is not continuous at $x=-1$.

- $f(x)$ has a VA at $x=-1 \to$ infinite = non-removable discontinuity.

#6 $f(x) = \begin{cases} -2x & ; x < 3 \\ -x^2+8x-16; & x \geq 3 \end{cases}$

- $3 \in D_f$

- $\lim_{x \to 3} f(x) = DNE$

- $\lim_{x \to 3^-} f(x) = -2(3) = -6$

- $\lim_{x \to 3^+} f(x) = -(3)^2+8(3)-16$
$= -1$

$\therefore f(x)$ is <u>not</u> continuous at $x=3$
because $\lim_{x \to 3} f(x) = DNE$.

$\therefore f(x)$ has a jump = non-removable discontinuity at $x=3$ because $\lim_{x \to 3^-} f(x) \neq \lim_{x \to 3^+} f(x)$.

#7 $f(x) = \dfrac{-x}{2x^2+2x+1}$

NPVs: $2x^2+2x+1=0$

$$x = \frac{-2 \pm \sqrt{2^2-4(2)(1)}}{2(2)}$$

no $\mathbb{R}$ sol

no NPVs

$\therefore$ f(x) is a continuous function for all $\mathbb{R}$ numbers.

#8 $f(x) = \dfrac{x}{x^2+6x+9} = \dfrac{x}{(x+3)(x+3)}$

NPV: $x=-3$

f(x) has a vertical asymptote at $x=-3$

$\rightarrow$ infinite = non-removable discontinuity.

$\therefore$ f(x) is continuous at every point of its domain

$\Rightarrow$ f(x) is a continuous function

$\Rightarrow$ f(x) is not continuous at $x=-3$

$\Rightarrow$ f(x) is not continuous on an interval around $x=-3$.

#9 $f(x) = \dfrac{x^2+4x+3}{x+3} = \dfrac{\cancel{(x+3)}(x+1)}{\cancel{(x+3)}} = x+1$

NPVs: $x=-3$

$\therefore$ f(x) is a continuous function

but f(x) has a point = removable discontinuity at $x=-3$

f(x) is not continuous on an interval around $x=-3$

#10 $f(x) = \dfrac{x}{x^2-4x} = \dfrac{x}{x(x-4)} = \dfrac{1}{x-4}$

NPVs:

$x=0$

$x=4$

- $f(x)$ is a continuous function.
- $f(x)$ is not continuous at $x=0$ → a point discont. = removable
  - at $x=4$ → infinite discont. = non-removable
- $f(x)$ is not continuous on intervals that contain $x=0$ or $x=4$ or both.

#11 $f(x) = \begin{cases} x+4 & , x \le -2 \\ -2x-11 & , x > -2 \end{cases}$

- $\lim_{x \to -2^+} f(x) = -2(-2) - 11 = -7$
- $\lim_{x \to -2^-} f(x) = -2+4 = 2$
- $\lim_{x \to -2} f(x) = \text{DNE}$
- $\therefore f(x)$ is not a continuous function

  $f(x)$ has a jump = non-removable disc. at $x=-2$.

- $f(x)$ is not continuous at $x=-2$ and not continuous on an interval around 2.

#12 $f(x) = \dfrac{x+7}{x^2+3x} = \dfrac{(x+7)}{x(x+3)}$ NDVs: $x=0$ $x=-3$

- $f(x)$ is a continuous function
- $f(x)$ is not continuous at $\left[\begin{matrix} x=0 \\ x=-3 \end{matrix}\right\}$ infinite = non-removable discontinuity
- $f(x)$ is not continuous on intervals containing $x=0$, $x=-3$, or both.

#13 $f(x) = \begin{cases} x & x \neq 4 \\ 2 & x=4 \end{cases}$ } point discontinuity

- $f(x)$ is continuous on $(-\infty, 4)$ and $(4, \infty)$.

#14. $f(x) = \begin{cases} -2 & ; x < 3 \\ -2x+6 & ; x \geq 3 \end{cases}$

- $\lim\limits_{x \to 3^+} f(x) = -2(3)+6 = 0$, $\lim\limits_{x \to 3^-} f(x) = -2$

$\Rightarrow$ jump discontinuity

$\therefore$ $f(x)$ is continuous on $(-\infty, 3) \cup [3, \infty)$.

#15 $f(x) = \dfrac{x-1}{x^2-4x+3} = \dfrac{\cancel{(x-1)}}{(x-3)\cancel{(x-1)}} = \dfrac{1}{x-3}$

NPVs:
$x=3$
$x=1$

- PD at $x=1$
- VA at $x=3$

$\therefore f(x)$ is continuous at : $(-\infty,1) \cup (1,3) \cup (3,\infty)$.

#16 $f(x) = \dfrac{x^2}{2} + 4x + 10$ No NPVs

$\therefore f(x)$ is continuous on $(-\infty, \infty)$.

#17 $f(x) = -x^2 - 4x + 2$ No NPVs

$\therefore f(x)$ is continuous on $(-\infty,\infty)$.

#18 $f(x) = -\dfrac{(x-2)}{x^2-3x+2} = -\dfrac{\cancel{(x-2)}}{\cancel{(x-2)}(x-1)} = \dfrac{-1}{x-1}$

NPVs : $x=2$, $x=1$

$\therefore f(x)$ is continuous at $(-\infty,1) \cup (1,2) \cup (2,\infty)$.

PD at $x=2$
VA at $x=1$

#21 There are many possible answers

VA at $x = 100$

$$f(x) = \frac{1}{x-100} \; ; \; g(x) = \frac{-1}{(x-100)^2} \; ; \; \ldots$$

#21 there are many possible answers

$(-\infty, 0) \cup (0,1) \cup (1, \infty)$

$\rightarrow$ discontinuities at $x=0$, $x=1$

$$f(x) = \frac{x+2}{(x-1)(x)} = \frac{x+2}{x^2-x}$$

$$g(x) = \frac{x^2-x}{x^2-x}$$

$$h(x) = \frac{5}{x^2-x}$$