C 12

# Continuity at a Point

Interior point: A function y=f(x) is continuous at an interior point c of its domain if

$$\lim_{x \to c} f(x) = f(c)$$

End point: A function y=f(x) is continuous at a left endpoint a or is continuous at a right endpoint b of its domain if

$$\lim_{x \to a^+} f(x) = f(a) \text{ and } \lim_{x \to b^-} f(x) = f(b), \text{ respectively}$$

- Recall: A continuous function is a function that is continuous at every point of its domain. A continuous function is not always continuous on every interval.

**Properties of continuous functions:**

- If f(x) and g(x) are both continuous at x=c, then their sum, difference, product, constant multiple and quotient (provided that g(c) is not zero) are all continuous at x=c.

- If g(x) is continuous at x=c and f(x) is continuous at g(c), then the composite function f(g(x)) is continuous at x=c.

**To determine whether a function is continuous at x=c the following must be true:**

1. **The given function is defined at x=c.**
2. **The two-sided limit of the function exists as x approaches c.**
3. **The value of the function at x=c is the same as the two-sided limit as x approaches c.**

# The Intermediate Value Theorem (IVT) for Continuous Functions

A function y=f(x) that is continuous on a closed interval [a,b] takes on every value between f(a) and f(b). In other words if $y_0$ is between f(a) and f(b), then $y_0$ = f(c) for some c in [a,b].

## Floor Function

## Ceiling Function

# Average Rate of Change

- Given by the slope of a secant line.
- The average rate of change of f(x) over the interval [a,b] is given by:

# Instantaneous Rate of Change

- Given by the slope of a tangent line.
- The instantaneous rate of change of f(x) at a point P (a,f(a)) is the slope of the curve at this point and it is identical to the slope of the tangent line to f(x) at point P.
- This slope m is given by:

Provided that the limit exists.