CALCULUS 12

CONNECTING f' and f'' TO THE GRAPH OF f

THEOREM: FIRST DERIVATIVE TEST FOR LOCAL EXTREMA

The following test applies to a continuous function f(x).

At a critical point c:

1. If f' changes sign from positive to negative at c (f'>0 for all x<c and f'<0 for all x>c), then f has a local **maximum** at c.

f' (c) = 0	f' (c) does not exist

2. If f' changes sign from negative to positive at c (f'< cfor all x<c and f'> 0 for all x>c), then f has a local **minimum** at c.

f' (c) = 0	f' (c) does not exist

3. If f' does not change sign at c (f' has the same sign on both sides of c), then f has no local extreme value at c.

f' (c) = 0	f' (c) does not exist

- **At the left endpoint a:** if f'<0 (f'>0) for all x> a, then f has a local maximum (minimum) value at a.

f' (a) = 0	f' (a) does not exist

- **At the right endpoint b :** if f'<0 (f'>0) for all x< b, then f has a local minimum (maximum) value at b.

f' (b) = 0	f' (b) does not exist

Example: Use the first derivative test to identify all local extrema of $y = -x^3+12x-5$

Note: y is a polynomial function that is continuous and differentiable on its entire domain. So the only critical points are those where y has a horizontal tangent.

Step 1:Find dy/dx and the points where it is zero.

Step 2: Use these points to split a number line into intervals. Within each interval identify whether the dy/dx is positive or negative.

Step 3: Identify whether the critical points yield a local maximum or minimum.

Step 4. Sketch the graph of y to check your work.

CONCAVITY

DEFINTION: CONCAVITY

The graph of a differentiable function $y = f(x)$ is

1. Concave up on an open interval I if y' is increasing on I.

2. Concave down on an open interval I if y' is decreasing on I.

Examples:

- **Concavity Test: The graph of a twice-differentiable function $y = f(x)$ is**
  1. concave up on any interval where $y'' > 0$
  2. concave down on any interval where $y'' < 0$

Example: Identify intervals on which $y = 2x^3 - x^2 - 5x$ is concave up and concave down.

POINTS OF INFLECTION

DEFINITION: a point where the graph of a function has a tangent line and where the concavity changes, is a point of inflection.

- A point of inflection can be either at the point where the second derivative zero or at the point where the second derivative does not exist.
- Not all points that have a zero second derivative are inflection points.
- An inflection point implies the change in the sign of the second derivative.

Examples:

SECOND DERIVATIVE TEST FOR LOCAL EXTREMA

1. If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at $x = c$.

2. If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at $x = c$.

Note: this test fails if $f''(c) = 0$ or when $f''(c)$ fails to exists and the first derivative test must be used instead.

Copy the chart from p 213:

Hw: p 215 #1-24