

Completing the Square

KEY

1) $a^2 + 2a - 3 = 0$

$y = a^2 + 2a - 3$

$y = (a+1)^2 - 4$

$\cdot \frac{2}{2} = 1$

$\cdot 1^2 = 1$

2) $a^2 - 2a - 8 = 0$

$y = a^2 - 2a - 8$

$y = a^2 - 2a + 1 - 1 - 8$

$y = (a-1)^2 - 9$

$\cdot \frac{-2}{2} = -1$

$\cdot (-1)^2 = 1$

3) $p^2 + 16p - 22 = 0$

$y = p^2 + 16p - 22$

$y = (p+8)^2 - 86$

$\cdot \frac{16}{2} = 8$

$\cdot 8^2 = 64$

4) $k^2 + 8k + 12 = 0$

$y = k^2 + 8k + 12$

$y = k^2 + 8k + 16 - 16 + 12$

$y = (k+4)^2 - 4$

$\cdot \frac{8}{2} = 4$

$\cdot 4^2 = 16$

5) $r^2 + 2r - 33 = 0$

$y = r^2 + 2r - 33$

$y = r^2 + 2r + 1 - 1 - 33$

$y = (r+1)^2 - 34$

$\cdot \frac{2}{2} = 1$

$\cdot 1^2 = 1$

6) $a^2 - 2a - 48 = 0$

$y = a^2 - 2a - 48$

$y = (a-1)^2 - 49$

$\cdot \frac{-2}{2} = -1$

$\cdot (-1)^2 = 1$

7) $m^2 - 12m + 26 = 0$

$y = m^2 - 12m + 26$

$y = m^2 - 12m + 36 - 36 + 26$

$y = (m-6)^2 - 10$

$\cdot \frac{-12}{2} = -6$

$\cdot (-6)^2 = 36$

8) $x^2 + 12x + 20 = 0$

$y = x^2 + 12x + 20$

$y = (x+6)^2 - 16$

$\cdot \frac{12}{2} = 6$

$\cdot 6^2 = 36$

9) $k^2 - 8k - 48 = 0$

$y = k^2 - 8k - 48$

$y = k^2 - 8k + 16 - 16 - 48$

$y = (k-4)^2 - 64$

$\cdot \frac{-8}{2} = -4$

$\cdot (-4)^2 = 16$

10) $p^2 + 2p - 63 = 0$

$y = p^2 + 2p - 63$

$y = (p+1)^2 - 64$

$\cdot \frac{2}{2} = 1$

$\cdot 1^2 = 1$

11) $m^2 + 2m - 48 = -6$

$y = m^2 + 2m - 42$

$y = m^2 + 2m + 1 - 1 - 42$

$y = (m+1)^2 - 43$

$\cdot \frac{2}{2} = 1$

$\cdot 1^2 = 1$

12) $p^2 - 8p + 21 = 6$

$y = p^2 - 8p + 15$

$y = p^2 - 8p + 16 - 16 + 15$

$y = (p-4)^2 - 1$

$\cdot \frac{-8}{2} = -4$

$\cdot (-4)^2 = 16$

$$13) m^2 + 10m + 14 = -7 \quad \cdot \frac{10}{2} = 5$$

$$y = m^2 + 10m + 21 \quad \cdot 5^2 = 25$$

$$y = m^2 + 10m + 25 - 25 + 21$$

$$y = (m + 5)^2 - 4$$

$$14) v^2 - 2v = 3 \quad \cdot \frac{-2}{2} = -1$$

$$y = v^2 - 2v - 3 \quad \cdot (-1)^2 = 1$$

$$y = v^2 - 2v + 1 - 1 - 3$$

$$y = (v - 1)^2 - 4$$

$$15) 5v^2 - 21 = 10v \quad \cdot \frac{-2}{2} = -1$$

$$y = 5v^2 - 10v - 21 \quad \cdot (-1)^2 = 1$$

$$y = 5(v^2 - 2v) - 21$$

$$y = 5(v^2 - 2v + 1 - 1) - 21$$

$$y = 5(v - 1)^2 - 26$$

$$16) 4v^2 + 16v = 65 \quad \cdot \frac{16}{2} = 8$$

$$y = 4v^2 + 16v - 65 \quad \cdot (2)^2 = 4$$

$$y = 4(v^2 + 4v) - 65$$

$$y = 4(v^2 + 4v + 4 - 4) - 65$$

$$y = 4(v + 2)^2 - 81$$

$$17) 7b^2 - 14b - 56 = 0 \quad \cdot \frac{-2}{2} = -1$$

$$y = 7(b^2 - 2b) - 56 \quad \cdot (-1)^2 = 1$$

$$y = 7(b^2 - 2b + 1 - 1) - 56$$

$$y = 7(b - 1)^2 - 63$$

$$18) 2n^2 + 12n + 10 = 0 \quad \cdot \frac{12}{2} = 6$$

$$y = 2(n^2 + 6n) + 10 \quad \cdot (3)^2 = 9$$

$$y = 2(n^2 + 6n + 9 - 9) + 10$$

$$y = 2(n + 3)^2 - 8$$

$$19) n^2 + 13n + 22 = 7 \quad \cdot \frac{13}{2}$$

$$y = n^2 + 13n + 15 \quad \cdot \left(\frac{13}{2}\right)^2 = \frac{169}{4}$$

$$y = n^2 + 13n + \frac{169}{4} - \frac{169}{4} + 15$$

$$y = \left(n + \frac{13}{2}\right)^2 - \frac{109}{4}$$

$$20) 5n^2 + 19n - 68 = -2 \quad \cdot \frac{19}{5} = \frac{19}{5}$$

$$y = 5n^2 + 19n - 66 \quad \cdot \left(\frac{19}{10}\right)^2 = \frac{361}{100}$$

$$y = 5\left(n^2 + \frac{19}{5}n\right) - 66$$

$$y = 5\left(n^2 + \frac{19}{5}n + \frac{361}{100} - \frac{361}{100}\right) - 66$$

$$y = 5\left(n + \frac{19}{10}\right)^2 - \frac{8405}{100}$$

$$21) r^2 - 9r - 38 = -9 \quad \cdot \frac{-9}{2}$$

$$y = r^2 - 9r - 29 \quad \cdot \left(\frac{-9}{2}\right)^2 = \frac{81}{4}$$

$$y = r^2 - 9r + \frac{81}{4} - \frac{81}{4} - 29$$

$$y = \left(r - \frac{9}{2}\right)^2 - \frac{197}{4}$$

$$22) 3x^2 + 20x + 36 = 4 \quad \cdot \frac{20}{3} = \frac{20}{3}$$

$$y = 3x^2 + 20x + 32 \quad \cdot \left(\frac{20}{6}\right)^2 = \frac{400}{9}$$

$$y = 3\left(x^2 + \frac{20}{3}x\right) + 32$$

$$y = 3\left(x^2 + \frac{20}{3}x + \frac{400}{36} - \frac{400}{36}\right) + 32$$

$$y = 3\left(x + \frac{10}{3}\right)^2 - \frac{4}{3}$$

$$24) n^2 + 19n + 66 = 6 \quad \cdot \frac{19}{2}$$

$$23) x^2 + 7x - 45 = 7 \quad \cdot \frac{7}{2}$$

$$y = x^2 + 7x - 52 \quad \cdot \left(\frac{7}{2}\right)^2 = \frac{49}{4}$$

$$y = x^2 + 7x + \frac{49}{4} - \frac{49}{4} - 52$$

$$y = \left(x + \frac{7}{2}\right)^2 - \frac{257}{4}$$

$$y = n^2 + 19n + 60 \quad \cdot \left(\frac{19}{2}\right)^2 = \frac{361}{4}$$

$$y = n^2 + 19n + \frac{361}{4} - \frac{361}{4} + 60$$

$$y = \left(n + \frac{19}{2}\right)^2 - \frac{121}{4}$$