# Differentiability

#1 $y = -x^2 + 3$

$y' = -2x$

$y'' = -2$

#2 $y = x^2 + x + 8$

$y' = 2x + 1$

$y'' = 2$

#3 $s = 5t^3 - 3t^5$

$s' = 15t^2 - 15t^4$

$s'' = 30t - 60t^3$

#4 $w = 3z^7 - 7z^3 + 21z^2$

$w' = 21z^6 - 21z^2 + 42z$

$w'' = 126z^5 - 42z + 42$

#5 $y = \frac{4x^3}{3} - x$

$y = \frac{4}{3}x^3 - x$

$y' = \frac{12}{3}x^2 - 1 = 4x^2 - 1$

$y'' = 8x$

#6 $y = \frac{x^3}{3} + \frac{x^2}{2} + \frac{x}{4} = \frac{1}{3}x^3 + \frac{1}{2}x^2 + \frac{1}{4}$

$y' = \frac{3}{3}x^2 + \frac{2}{2}x + \frac{1}{4}$

$= x^2 + x + \frac{1}{4}$

$y'' = 2x + 1$

#7 $w = 3z^{-2} - \frac{1}{z}$

$w = 3z^{-2} - z^{-1}$

$w' = -6z^{-3} - (-1)(z)^{-2}$

$w' = -6z^{-3} + z^{-2}$

$w'' = 18z^{-4} - 2z^{-3}$

#8 $s = -2t^{-1} + \frac{4}{t^2}$

$s = -2t^{-1} + 4t^{-2}$

$s' = +2t^{-2} - 8t^{-3}$

$s'' = -4t^{-3} + 24t^{-4}$

#9 $y = 6x^2 - 10x - 5x^{-2}$

$y' = 12x - 10 + 10x^{-3}$

$y' = 12 - 30x^{-4}$

#10 $y = 4 - 2x - x^{-3}$

$y' = -2 + 3x^{-4}$

$y'' = -12x^{-5}$

#11 $r = \frac{1}{3s^2} - \frac{5}{2s}$

$r = \frac{1}{3}s^{-2} - \frac{5}{2}s^{-1}$

$r' = \frac{-2}{3}s^{-3} + \frac{5}{2}s^{-2}$

$r'' = \frac{6}{3}s^{-4} - \frac{10}{2}s^{-3}$

$r'' = 2s^{-4} - 5s^{-3}$

#12 $r = \frac{12}{\theta} - \frac{4}{\theta^3} - \frac{1}{\theta^4}$

$r = 12\theta^{-1} - 4\theta^{-3} - 1\theta^{-4}$

$r' = -12\theta^{-2} + 12\theta^{-4} + 4\theta^{-5}$

$r'' = 24\theta^{-3} - 48\theta^{-5} - 20\theta^{-6}$

#13 $y = (3-x^2)(x^3 - x + 1)$

$y' = -2x(x^3 - x + 1) + (3-x^2)(3x^2 - 1)$

$= -2x^4 + 2x^2 - 2x + 9x^2 - 3 - 3x^4 + x^2$

$= -5x^4 + 12x^2 - 2x - 3$

$y = 3x^3 - 3x + 3 - x^5 + x^3 - x^2$

$= 4x^3 - x^5 - x^2 - 3x + 3$

$y' = -5x^4 + 12x^2 - 2x - 3$

#14 $y = (x-1)(x^2+x+1)$

$y' = 1(x^2+x+1) + (x-1)(2x+1)$

$y' = (x^2+x+1) + (2x^2+x-2x-1)$

$y' = 3x^2$

$y = (x-1)(x^2+x+1)$

$= x^3 + x^2 + x - x^2 - x - 1$

$= x^3 - 1$

$y' = 3x^2$

#15 $y = (x^2+1)(x+5+\frac{1}{x})$

$y' = 2x(x+5+\frac{1}{x}) + (x^2+1)(1-x^{-2})$

$y' = 2x^2 + 10x + 2 + x^2 + 1 - x^{-2} - 1$

$y' = 3x^2 + 10x - x^{-2} + 2$

$y = (x^2+1)(x+5+x^{-1})$

$y = x^3 + 5x^2 + x + x + 5 + x^{-1}$

$y = x^3 + 5x^2 + x^{-1} + 2x + 5$

$y' = 3x^2 + 10x + 2 - x^{-2}$

$= 3x^2 + 10x - x^{-2} + 2$

$x^2(-x^{-2}) = -1x^{2-2} = -1x^0 = -1$

#16 $y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$

$y' = \left(1 + (-1)x^{-2}\right)\left(x - \frac{1}{x} + 1\right) + \left(x + \frac{1}{x}\right)\left(1 + x^{-2}\right)$

$y' = \left(1 - \frac{1}{x^2}\right)\left(x - \frac{1}{x} + 1\right) + \left(x + \frac{1}{x}\right)\left(1 + \frac{1}{x^2}\right)$

$y' = x - \frac{1}{x} + 1 - \frac{1}{x} + \frac{1}{x^3} - \frac{1}{x^2} + x + \frac{1}{x} + \frac{1}{x} + \frac{1}{x^3}$

$y' = 2x + 1 - \frac{1}{x^2} + \frac{2}{x^3}$

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$y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$

$y = x^2 - 1 + x + 1 - \frac{1}{x^2} + \frac{1}{x}$

$y = x^2 + x - x^{-2} + x^{-1}$

$y' = 2x + 1 + 2x^{-3} - x^{-2}$

$y = 2x + 1 + \frac{2}{x^3} - \frac{1}{x^2}$

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#17 $y = \frac{2x+5}{3x-2}$

$y' = \frac{2(3x-2) - [(2x+5)(3)]}{(3x-2)^2}$

$y' = \frac{6x - 4 - [6x + 15]}{(3x-2)^2}$

$y' = \frac{19}{(3x-2)^2}$

#18 $z = \frac{2x+1}{x^2-1}$

$$z' = \frac{2(x^2-1) - [(2x+1)(2x)]}{(x^2-1)^2}$$

$$z' = \frac{2x^2-2-[4x^2+2x]}{(x^2-1)^2}$$

$$z' = \frac{-2x^2+2x-2}{(x^2-1)^2}$$

$$z' = \frac{-2(x^2-x+1)}{(x^2-1)^2}$$

#19 $g(x) = \frac{x^2-4}{x+0.5}$

$$g'(x) = \frac{2x(x+0.5) - [(x^2-4)(1)]}{(x+0.5)^2}$$

$$g'(x) = \frac{2x^2+x-x^2+4}{(x+0.5)^2}$$

$$g'(x) = \frac{x^2+x+4}{(x+0.5)^2}$$

#20 $f(t) = \dfrac{t^2-1}{t^2+t-2}$

$f'(t) = \dfrac{2t(t^2+t-2) - [(t^2-1)(2t+1)]}{(t^2+t-2)^2}$

$f'(t) = \dfrac{2t^3+2t^2-4t - [2t^3+t^2-2t-1]}{(t^2+t-2)^2}$

$f'(t) = \dfrac{t^2-2t+1}{(t^2+t-2)^2}$

$f'(t) = \dfrac{(t-1)^2}{(t^2+t-2)^2}$

#21 $x = (1-t)(1+t^2)^{-1}$

$x = \dfrac{1-t}{1+t^2}$

$x' = \dfrac{-1(1+t^2) - [(1-t)(2t)]}{(1+t^2)^2}$

$x' = \dfrac{-1-t^2-2t+2t^2}{(1+t^2)^2}$

$x' = \dfrac{t^2-2t-1}{(1+t^2)^2}$

Using Chain Rule:

$x' = (-1)(1+t^2)^{-1} + (1-t)(-1)(1+t^2)^{-2}(2t)$

$x' = \dfrac{-1}{1+t^2} - \dfrac{2t-2t^2}{(1+t^2)^2}$

$x' = \dfrac{-1-t^2-(2t-2t^2)}{(1+t^2)^2}$

$x' = \dfrac{t^2-2t-1}{(1+t^2)^2}$

#22 $w = (2x-7)^{-1}(x+5) = \dfrac{x+5}{2x-7}$

$w' = \dfrac{1(2x-7) - [(x+5)(2)]}{(2x-7)^2}$

$w' = \dfrac{2x-7-2x-10}{(2x-7)^2}$

$w' = \dfrac{-17}{(2x-7)^2}$

By chain rule:

$w' = (-1)(2x-7)^{-2}(2)(x+5) + (2x-7)^{-1}(1)$

$w' = \dfrac{-1}{(2x-7)^2}(2x+10) + \dfrac{1}{(2x-7)}$

$w' = \dfrac{-2x-10}{(2x-7)^2} + \dfrac{2x-7}{(2x-7)^2}$

$w' = \dfrac{-17}{(2x-7)^2}$

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#23 $f(s) = \dfrac{\sqrt{s}-1}{\sqrt{s}+1}$

$f(s) = \dfrac{s^{\frac{1}{2}}-1}{s^{\frac{1}{2}}+1}$

$f'(s) = \dfrac{\left(\frac{1}{2}s^{-\frac{1}{2}}\right)\left(s^{\frac{1}{2}}+1\right) - \left[\left(s^{\frac{1}{2}}-1\right)\left(\frac{1}{2}s^{-\frac{1}{2}}\right)\right]}{\left(s^{\frac{1}{2}}+1\right)^2}$

$f'(s) = \dfrac{\frac{1}{2} + \frac{1}{2}s^{-\frac{1}{2}} - \left(\frac{1}{2} - \frac{1}{2}s^{-\frac{1}{2}}\right)}{\left(s^{\frac{1}{2}}+1\right)^2}$

$= \dfrac{s^{-\frac{1}{2}}}{\left(s^{\frac{1}{2}}+1\right)^2} = \dfrac{\frac{1}{\sqrt{s}}}{\left(\sqrt{s}+1\right)^2} = \dfrac{1}{\sqrt{s}\left(\sqrt{s}+1\right)^2}$

#24 $u = \dfrac{5x+1}{2\sqrt{x}}$

$u = \dfrac{5x+1}{2x^{\frac{1}{2}}}$

$u = (5x+1)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$

$u' = 5\left(\frac{1}{2}x^{-\frac{1}{2}}\right) + (5x+1)\left(-\frac{1}{4}x^{-\frac{3}{2}}\right)$

$u' = \dfrac{5}{2\sqrt{x}} + \left(\frac{-5}{4}x^{1-\frac{3}{2}}\right) - \dfrac{1}{4x^{\frac{3}{2}}}$

$u' = \dfrac{5}{2\sqrt{x}} - \dfrac{5}{4\sqrt{x}} - \dfrac{1}{4x\sqrt{x}}$

$u' = \dfrac{10x - 5\cdot x - 1}{4x\sqrt{x}}$

$u' = \dfrac{5x-1}{4x\sqrt{x}}$

$u' = \dfrac{\sqrt{x}(5x-1)}{4x^2}$

#25

$$r = \frac{1+x-4\sqrt{x}}{x}$$

$$r' = \frac{\left(1+4\left(\frac{1}{2}\right)x^{-\frac{1}{2}}\right)x - \left[(1+x-4\sqrt{x})(1)\right]}{x^2}$$

$$r' = \frac{x+2x^{\frac{1}{2}}-1-x+4x^{\frac{1}{2}}}{x^2}$$

$$r' = \frac{+2x^{\frac{1}{2}}-1}{x^2}$$

$$r' = \frac{+2\sqrt{x}-1}{x^2}$$

$$r' = \frac{+2x^{\frac{1}{2}}}{x^2} - \frac{1}{x^2}$$

$$r' = +2x^{-\frac{3}{2}} - \frac{1}{x^2}$$

#26 $r = 2\left(\frac{1}{\sqrt{\theta}} + \sqrt{\theta}\right)$

$$r = 2\left(\theta^{-\frac{1}{2}} + \theta^{\frac{1}{2}}\right)$$

$$r' = 2\left[\left(-\frac{1}{2}\right)\left(\theta^{-\frac{3}{2}}\right) + \left(\frac{1}{2}\right)\left(\theta^{-\frac{1}{2}}\right)\right]$$

$$r' = -\theta^{-\frac{3}{2}} + \theta^{-\frac{1}{2}}$$

$$r' = \frac{-1}{\theta^{\frac{3}{2}}} + \frac{1}{\theta^{\frac{1}{2}}}$$

#27 $y = \dfrac{1}{(x^2-1)(x^2+x+1)}$

$$y' = \frac{-[2x(x^2+x+1)+(x^2-1)(2x+1)]}{(x^2-1)^2(x^2+x+1)^2}$$

$$y' = \frac{-[2x^3+2x^2+2x+2x^3+x^2-2x-1]}{(x^2-1)^2(x^2+x+1)^2}$$

$$y' = \frac{-[4x^3+3x^2-1]}{(x^2-1)^2(x^2+x+1)^2}$$

$$y' = \frac{-4x^3-3x^2+1}{(x^2-1)^2(x^2+x+1)^2}$$

#28 $y = \dfrac{(x+1)(x+2)}{(x-1)(x-2)}$

$y = \dfrac{x^2+2x+x+2}{x^2-2x-x+2}$

$y = \dfrac{x^2+3x+2}{x^2-3x+2}$

$y' = \dfrac{(2x+3)(x^2-3x+2) - [(x^2+3x+2)(2x-3)]}{(x-1)^2(x-2)^2}$

$y' = \dfrac{2x^3-6x^2+4x+3x^2-9x+6-[2x^3-3x^2+6x^2-9x+4x-6]}{(x-1)^2(x-2)^2}$

$y' = \dfrac{-6x^2+12}{(x-1)^2(x-2)^2} = \dfrac{-6(x^2-2)}{(x-1)^2(x-2)^2}$

#29 $y = \frac{x^4}{2} - \frac{3}{2}x^2 - x$

$y = \frac{1}{2}x^4 - \frac{3}{2}x^2 - x$

$y' = 2x^3 - 3x - 1$

$y'' = 6x^2 - 3$

$y''' = 12x$

$y^{(4)} = 12$

#30 $y = \frac{x^5}{120}$

$y = \frac{1}{120}x^5$

$y' = \frac{5}{120}x^4 = \frac{1}{24}x^4$

$y'' = \frac{4}{24}x^3 = \frac{1}{6}x^3$

$y''' = \frac{3}{6}x^2 = \frac{1}{2}x^2$

$y^{(4)} = x$

$y^{(5)} = 1$